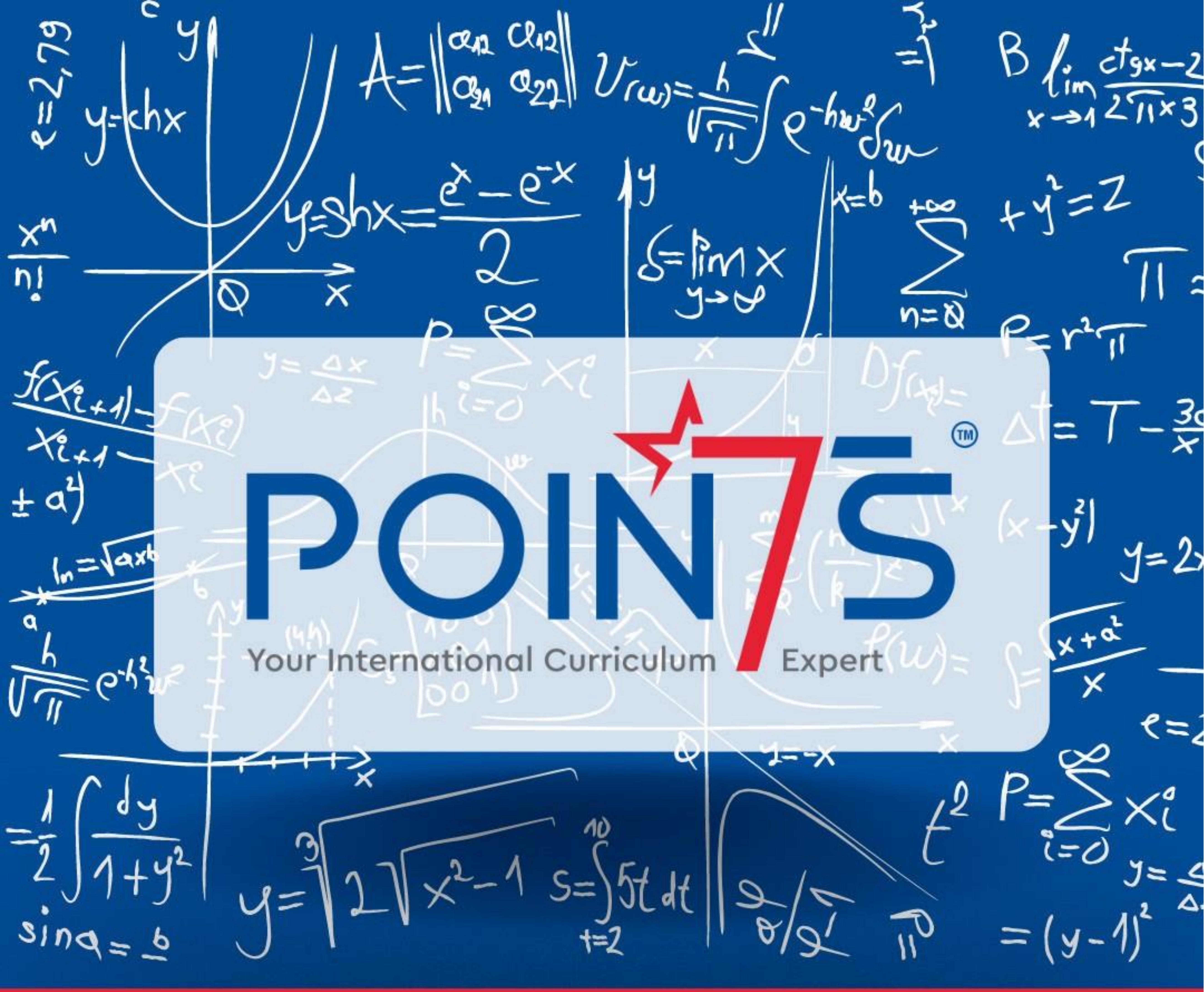

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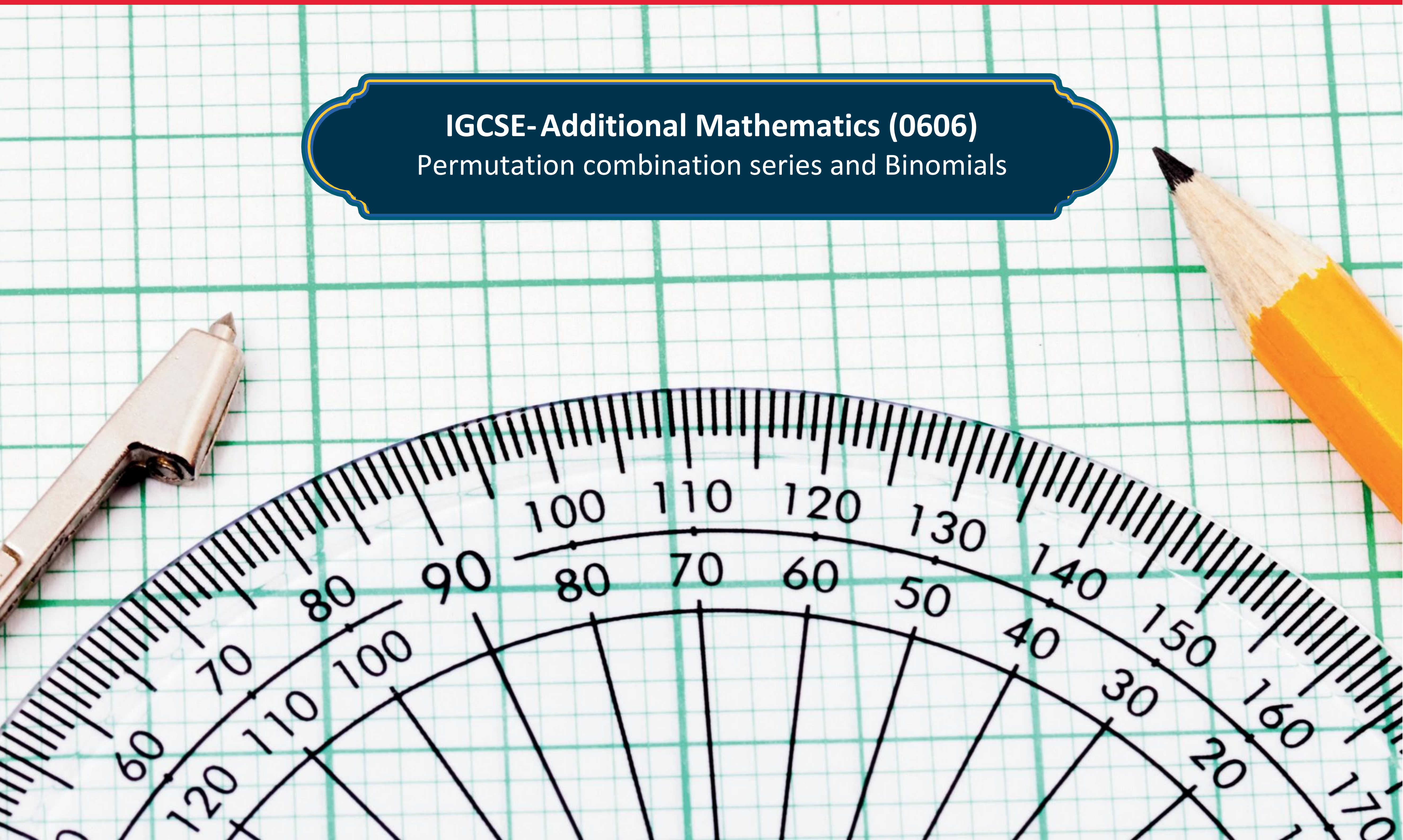
MATH TOPICAL WORKSHEETS

IGCSE- Additional Mathematics (0606)
Permutation combination series and Binomials



1

Substitute $n = n$ and $r = 4$ into the formula ${}^nC_r$

$$\frac{n!}{(n-4)!4!}$$

Substitute $n = (n+1)$ and $r = 5$ into the formula ${}^nC_r$

$$\frac{(n+1)!}{((n+1)-5)!5!}$$

Substitute into the given equation.

$$45 \times \frac{n!}{(n-4)!4!} = \frac{(n+1)(n+1)!}{((n+1)-5)!5!}$$

[1]

Simplify.

$$\frac{45n!}{24(n-4)!} = \frac{(n+1)(n+1)!}{120(n-4)!}$$

Multiply both sides by $24(n-4)!$

$$45n! = \frac{24(n+1)(n+1)!(n-4)!}{120(n-4)!}$$

Cancel $(n-4)!$ from top and bottom.

$$45 = \frac{24(n+1)(n+1)!}{120n!}$$

$\frac{(n+1)!}{n!} = n+1$, simplify.

$$45 = \frac{(n+1)^2}{5}$$

Rearrange to achieve a quadratic equation.

$$n^2 + 2n - 224 = 0$$

[1]

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Factorise.

$$(n+16)(n-14)=0$$

[1]

Solve.

$n=-16$ and $n=14$

n cannot be negative.

$n = 14$ [1]

2a

i) To get from term to term, divide by -2 which is the same as multiplying by $-\frac{1}{2}$. So this is a geometric series with

$$a=4,\ r=-\frac{1}{2}$$

Use the formula $u_n = ar^{n-1}$ with $n = 20$ to work out the 20th term.

$$u_{20}=4\times\left(-\frac{1}{2}\right)^{20-1}$$

[1]

$$u_{20}=-\frac{1}{131072}$$ [1]

ii) The sum to infinity exists if the modulus of the common ratio is less than 1, i.e. $|r|<1$

$$|r|=\left|-\frac{1}{2}\right|=\frac{1}{2}<1$$

$|r|<1$ so the sum to infinity does exist [1]

Use the sum to infinity formula, $S_\infty=\frac{a}{1-r}$.

$$S_\infty=\frac{4}{1-\left(-\frac{1}{2}\right)}=\frac{8}{3}$$

$$S_\infty=\frac{8}{3}$$ [1]

2b

i) Use the information in the question to make 2 equations.

For the first equation, use "the tenth term ... is 15 times the second term".

$$u_{10} = 15 \times u_2$$

Use $u_n = a + (n-1)d$ for the tenth and second terms and set equal to each other.

$$a + (10-1)d = 15 \times (a + (2-1)d)$$
$$a + 9d = 15(a + d)$$

[1]

$$a + 9d = 15a + 15d$$
$$14a = -6d$$
$$a = -\frac{3}{7}d$$

For the second equation use "the sum of the first 6 terms ... is 87". "$S_n = \frac{1}{2}n(2a + (n-1)d)$"

$$S_6 = \frac{1}{2} \times 6(2a + 5d) = 87$$

[1]

$$3(2a + 5d) = 87$$
$$2a + 5d = 29$$

Solve the two equations simultaneously.

$$2\left(-\frac{3}{7}d\right) + 5d = 29$$

[1]

$$-\frac{6}{7}d + 5d = 29$$
$$\frac{29}{7}d = 29$$

$d = 7$ [1]

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ii) From (i), $d = 7$ and $a = -\frac{3}{7}d$. Find a.

$$a = -\frac{3}{7} \times 7 = -3$$

[1]

Use the formula $u_n = a + (n-1)d$ to set the nth term equal to 6990.

$$6990 = -3 + (n-1)7$$

[1]

$$6993 = 7n - 7$$
$$7000 = 7n$$

$n = 1000$ [1]

3

i) There are 8 digits to choose from, and we need a 5 digit number.

$$^{8}P_{5} = \frac{8!}{(8-5)!}$$

$$= 8 \times 7 \times 6 \times 5 \times 4$$

[1]

6720 [1]

ii) For the number to be greater than 60000, the first digit can be either 6, 7 or 8.

For the first digit, we have three options. For the other digits, there are seven options, of which we want four.

$$3 \times {}^{7}P_{4}$$

[1]

2520 [1]

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4a

This is a geometric progression with first term.

$$a = 3$$

Work out the common ratio by dividing the second term by the first.

$$r = \frac{2.4}{3}$$

$$r = 0.8$$

[1]

Use the sum of a geometric progression formula, $S_n = \frac{a(1 - r^n)}{1 - r}$, to calculate the sum of the first 8 terms.

$$S_8 = \frac{3(1 - 0.8^8)}{1 - 0.8}$$

[1]

$S_8 = 12.48$ [1]

12.5 is accepted

4b

From part (a) we know that $a = 3$, $r = 0.8$.

Substitute these values into the sum to infinity formula, $S_\infty = \frac{a}{1 - r}$.

$$S_\infty = \frac{3}{1 - 0.8}$$

15 [1]

4c

Calculate 95% of the sum to infinity using the result from part (b).

$$0.95 \times 15 = 14.25$$

Set up an inequality using the sum of the first n terms, $S_n = \frac{a(1-r^n)}{1-r}$.

$$\frac{a(1-r^n)}{1-r} > 14.25$$

Substitute the values $a = 3$, $r = 0.8$ from part (a).

$$\frac{3(1-0.8^n)}{1-0.8} > 14.25$$

[1]

Simplify the denominator.

$$\frac{3(1-0.8^n)}{0.2} > 14.25$$

Multiply both sides by 0.2.

$$3(1-0.8^n) > 2.85$$

Divide both sides by 3.

$$1-0.8^n > 0.95$$

Add 0.8^n to both sides and subtract 0.95.

$$0.05 > 0.8^n$$

[1]

Take logarithms of both sides.

$$\log_{10}(0.05) > \log_{10}(0.8^n)$$

Divide both sides by $\log_{10}(0.8)$. This is a negative value so reverse the inequality sign.

$$\frac{\log_{10}(0.05)}{\log_{10}(0.8)} < n$$

[1]

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Calculate.

$$n > 13.425...$$

n is an integer so write down the smallest integer that is greater than 13.425...

$n = 14$ [1]

5

Using the formula for the binomial expansion, finding the first three terms gives

$$\left(a^5 + \binom{5}{1}a^4bx + \binom{5}{2}a^3(bx)^2\right)(1 + x)$$

[1]

Simplifying gives

$$(a^5 + 5a^4bx + 10a^3b^2x^2)(1 + x)$$

[1]

Expanding the brackets

$$a^5 + 5a^4bx + 10a^3b^2x^2 + a^5x + 5a^4bx^2 + 10a^3b^2x^3$$

multiplying out brackets [1]

all terms correct [1]

Comparing to $32 - 208x + cx^2$ by comparing the constants we have

$$a^5 = 32$$

$a = 2$ [1]

Equating the coefficients of x we have

$$5a^4b + a^5 = -208$$

Substituting in $a = 2$

$$80b + 32 = -208$$

$$80b = -240$$

$b = -3$ [1]

Equating the coefficients of x^2 we have

$$10a^3b^2 + 5a^4b = c$$

Substituting in $a = 2$ and $b = -3$ we have

$$720 - 240 = c$$

$c = 480$ [1]

6

i) There are 6 digits to choose from, and we need a 5 digit number.

$$^6P_5 = \frac{6!}{(6-5)!}$$

$$= 6 \times 5 \times 4 \times 3 \times 2$$

720 [1]

ii) $\frac{2}{3}$ of the digits are odd.

$$720 \times \frac{2}{3}$$

480 [1]

iii) For the number to be greater than 60000 and odd, the first digit can be either 6, 8 or 9 and the last digit must be odd.

If the first digit is either a 6 or an 8, there are 4 options for the last digit $(1, 3, 5, 9)$.

$$4 \times (4 \times 3 \times 2)$$

$$= 96$$

This is the case when the first digit is 6 **or** when it is 8 (2 possibilities).

$$96 \times 2$$

$= 192$ numbers

[1]

If the first digit is a 9, there are only 3 options for the last digit $(1, 3, 5)$ because the 9 has already been used.

$$3\times(4\times3\times2)$$

$= 72$ numbers

[1]

If the first digit is 6, 8 or 9

$$192+72$$

264 [1]

7a

Using $u_n = a + (n-1)d$ write equations for the 7th and 10th terms of the arithmetic progression.

$$a+6d=158$$
$$a+9d=149$$

[1]

Solve the equations simultaneously.

$$\begin{array}{rr} & a+9d=149 \\ - & a+6d=158 \\ \hline & 3d=\quad -9 \end{array}$$

$d = -3$ [1]

Substitute into one of the original equations.

$$a+(6)(-3)=158$$

$a = 176$ [1]

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7b

Substitute the values found in part a into the formula to sum an arithmetic series.

$$S_n = \frac{n}{2}[2(176) + (n-1)(-3)]$$
$$S_n = \frac{n}{2}[352 - 3n + 3]$$
$$S_n = \frac{n}{2}[355 - 3n]$$

[1]

The sum of the progression needs to be negative.

$$\frac{n}{2}[355 - 3n] < 0$$

Solve - be careful, it's a quadratic inequality, so use a calculator or sketch the graph.

$n < 0$ and $n > 118.333...$

[1]

The value of n cannot be negative, so $n > 118.333...$ n has to be an integer

$n = 119$ [1]

8a

The sum of an arithmetic progression is $S_n = \frac{n}{2}[2a + (n-1)d]$. Write an expression for the sum of the first four terms.

$$S_4 = \frac{4}{2}[2a + (4-1)d]$$

[1]

$$2[2a + 3d] = 38$$
$$2a + 3d = 19$$

[1]

Similarly, write an expression for the sum of the first eight terms.

$$S_8 = \frac{8}{2}[2a + (8-1)d]$$
$$S_8 = 4[2a + 7d]$$

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We are told that the first four terms sum to 38, and the next four sum to 86. Therefore, the sum of the first 8 terms is the sum of 38 and 86.

$$4[2a + 7d] = 38 + 86$$

[1]

Expand and simplify.

$$2a + 7d = 31$$

Solve the equations simultaneously.

$$\begin{aligned} 2a + 7d &= 31 \\ - \quad 2a + 3d &= 19 \\ \hline 4d &= 12 \\ d &= 3 \end{aligned}$$

[1]

Substitute $d = 3$ into one of the original equations and solve for a.

$$\begin{aligned} 2a + 7(3) &= 31 \\ 2a &= 31 - 21 \\ a &= 5 \end{aligned}$$

$a = 5$ and $d = 3$ [1]

8b

Using the nth term formula for a geometric progression, $u_n = ar^{n-1}$, the second term is

$$ar^2 = 12$$

[1]

The sixth term is

$$ar^5 = -96$$

[1]

Solve the equations simultaneously - rearrange the first and substitute into the second.

$$a = \frac{12}{r^2}$$

$$\frac{12}{r^2}(r^5) = -96$$

Solve for r.

$$12r^3 = -96$$
$$r^3 = -8$$
$$r = -2$$

[1]

Substitute into either of the original equations to find a.

$$a(-2)^2 = 12$$
$$a = 3$$

[1]

The sum of a geometric progression is $S_n = \dfrac{a(1-r^n)}{1-r}$.

$$S_{10} = \frac{3(1-(-2)^{10})}{1-(-2)}$$

[1]

$S_{10} = -1023$ [1]

9

For there to be **at least** three women, there could either be two women and one man, or three women.

To work out the number of different teams that have two women and one man:

$$^4C_1 \times {}^5C_2$$

To work out the number of different teams that have three women:

$$^5C_3$$

Add together to work out the number of teams that have two women and one man **or** three women.

$$^4C_1 \times {}^5C_2 + {}^5C_3$$

[1]

50 [1]

10a

Substitute $n = 20$ into the formula $u_n = a + (n-1)d$

$$u_{20} = a + (20-1)d$$
$$= a + 19d$$

Substitute $n = 25$ into the formula $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{25} = \frac{25}{2}[2a + (25-1)d]$$
$$= \frac{25}{2}(2a + 24d)$$

Substitute the two expressions above into the 10% relationship from the question, $u_{20} = 0.1S_{25}$

$$a + 19d = 0.1 \times \frac{25}{2}(2a + 24d)$$

[M1]

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Make a (or d) the subject

$$a + 19d = \frac{5}{4}(2a + 24d)$$
$$a + 19d = \frac{5}{2}(a + 12d)$$
$$2(a + 19d) = 5(a + 12d)$$
$$2a + 38d = 5a + 60d$$
$$-3a = 22d$$
$$a = -\frac{22}{3}d$$

[A1]

Find an expression for the sum of the 20th and the 21st term using $u_n = a + (n-1)d$

$$(a + 19d) + (a + 20d)$$

Set this equal to 73

$$(a + 19d) + (a + 20d) = 73$$

[M1]

Simplify the equation above then substitute in $a = -\frac{22}{3}d$ to find d

$$a = -22$$

[A1]

Substitute $a = -22$ and $d = 3$ into $u_n = a + (n-1)d$ where $n = 8$ to find the 8th term

$$-22 + (8-1) \times 3$$

The 8th term is -1

[A1]

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10b

Use the formula $S_\infty = \frac{a}{1-r}$ to form an equation in a and r (equation 1)

$$\frac{a}{1-r} = 64 \quad (1)$$

[B1]

Use the formula $S_n = \frac{a(1-r^n)}{1-r}$ where $n = 7$ and $S_7 = \frac{127}{2}$ to form another equation in a and r (equation 2)

$$\frac{a(1-r^7)}{1-r} = \frac{127}{2} \quad (2)$$

[B1]

Make a the subject of equation 1 and substitute it into equation 2

$$\frac{64(1-r)(1-r^7)}{1-r} = \frac{127}{2}$$

$$\frac{64\cancel{(1-r)}(1-r^7)}{\cancel{1-r}} = \frac{127}{2}$$

$$64(1-r^7) = \frac{127}{2}$$

[M1]

Solve for r

$$1 - r^7 = \frac{127}{128}$$

$$1 - \frac{127}{128} = r^7$$

$$\frac{1}{128} = r^7$$

$$\sqrt[7]{\frac{1}{128}} = r$$

$$r = \frac{1}{2}$$

[A1]

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Use $u_n = ar^{n-1}$ to form a ratio between the 4th and 7th terms

$$ar^3 : ar^6$$

[M1]

Cancel the a and r^3 from both sides and substitute in $r = \frac{1}{2}$

$$1 : r^3$$
$$1 : \left(\frac{1}{2}\right)^3$$
$$1 : \frac{1}{8}$$

Write this in the form $m : 1$ by multiplying both sides by 8

$8 : 1$

[A1]

11a

(i)

Method 1

There are 7 possibilities for the first digit (3, 4, 5, 6, 7, 8 or 9)

There are 6 possible choices left for the second digit

There are 5 possible choices left for the third digit

There are 4 possible choices left for the fourth digit

$$7 \times 6 \times 5 \times 4$$

840

[B1]

Method 2

Choose 4 digits out of a possible 7 different digits (3, 4, 5, 6, 7, 8 or 9) with no repeats and where order does matter (i.e. use permutations, not combinations)

$${}^7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!}$$

840

[B1]

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(ii)

Method 1

Numbers less than 9000 has a first digit that is less than 9

So there are 6 possible choices for the first digit (3, 4, 5, 6, 7 or 8)

The 9 is allowed in the remaining digits

So there are 6 possible choices left for the second digit (out of 3, 4, 5, 6, 7, 8 and 9 but with one choice taken away for the first digit)

There are 5 possible choices left for the third digit

There are 4 possible choices left for the fourth digit

$$6 \times 6 \times 5 \times 4$$

[M1]

720

[A1]

Method 2

The same first digit as Method 1, then using ${}^{6}P_{3}$ to select the next 3 digits out of the remaining 6 digits (3, 4, 5, 6, 7, 8 and 9 but with one choice taken away for the first digit)

$$6 \times {}^{6}P_{3}$$

[M1]

720

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11b

Order does not matter in this question (i.e. use combinations, not permutations)

(i)

An equal number of each type means 2 horrors, 2 romances and 2 travel books

Out of 5 horror books select 2

$$^5C_2$$

Out of 6 romance books select 2

$$^6C_2$$

Out of 4 travel books select 2

$$^4C_2$$

Multiply the combinations together

$$^5C_2 \times {}^6C_2 \times {}^4C_2$$

[M1]

900

[A1]

(ii)

Method 1

Split into separate cases then add the separate cases together

Case 1: select 4 horror books out of 5, then 1 romance book out of 6, then 1 travel book out of 4

$$^5C_4 \times {}^6C_1 \times {}^4C_1$$

[M1]

Case 2: select 4 horror books out of 5, then 0 romance book out of 6, then 2 travel books out of 4

$$^5C_4 \times {}^6C_0 \times {}^4C_2$$

[M1]

Add these separate cases together

$$= {}^5C_4 \times {}^6C_1 \times {}^4C_1 + {}^5C_4 \times {}^6C_0 \times {}^4C_2$$
$$= 120 + 30$$

150

[A1]

Method 2

Out of 5 horror books select 4

$${}^5C_4$$

[M1]

Then combine the romance and travel books into 6 + 4 = 10 books and select the remaining 2 books from these

$${}^{10}C_2$$

But you don't want any of the cases with '2 romance + 0 travel' so subtract these combinations

$${}^{10}C_2 - {}^6C_2 \times {}^4C_0$$

[M1]

Multiply this by the selection of 4 horror books

$${}^5C_4\left({}^{10}C_2 - {}^6C_2 \times {}^4C_0\right)$$

150

[A1]

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12

Use the binomial theorem $(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + b^n$

$$(3-ax)^5 = 3^5 + \binom{5}{1}3^4(-ax) + \binom{5}{2}3^3(-ax)^2 + \ldots$$

$$(3-ax)^5 = 243 - 405ax + 270a^2x^2 + \ldots$$

1 mark for second and third term [2]

Set the first 3 terms equal to the expression given in the question:

$$b - 81x + cx^2 = 243 - 405ax + 270a^2x^2$$

Equate coefficients:

$$b = 243$$

[1]

$$-81 = -405a \text{ so } a = \frac{1}{5}$$

[1]

$$c = 270a^2 = 270 \times \left(\frac{1}{5}\right)^2 = \frac{54}{5}$$

$a = \frac{1}{5}$, $b = 243$, $c = \frac{54}{5}$ [1]

13a

Using the general equation for the nth term, $u_n = a + (n-1)d$ where a is the first term and d is the common difference, we will form two simultaneous equations in a and d. We can write the 5th term as

$$u_5 = a + (5-1)d$$
$$u_5 = a + 4d$$

and the 16th term as

$$u_{16} = a + (16-1)d$$
$$u_{16} = a + 15d$$

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The question states "the 5th term is equal to $\frac{1}{3}$ of the 16th term" so

$$a + 4d = \frac{1}{3}(a + 15d)$$

[1]

Simplify.

$$a + 4d = \frac{1}{3}a + 5d$$
$$\frac{2}{3}a = d$$

Use the information that "the sum of the 5th term and the 16th term is equal to 33".

$$(a + 4d) + (a + 15d) = 33$$

[1]

Simplify.

$$2a + 19d = 33$$

Substitute $d = \frac{2}{3}a$ into this equation.

$$2a + 19\left(\frac{2}{3}a\right) = 33$$

Solve for a.

$$\frac{44}{3}a = 33$$
$$a = \frac{99}{44}$$

Simplify.

$$a = \frac{9}{4}$$

[1]

Substitute $a = \frac{9}{4}$ into $\frac{2}{3}a = d$.

$$\frac{2}{3}\times\frac{9}{4}=d$$

$$d=\frac{18}{12}$$

Simplify.

$$d=\frac{3}{2}$$

[1]

We haven't finished! The question asks for the sum of the first 10 terms.

Use the formula $S_n=\frac{n}{2}(2a+(n-1)d)$.

$$S_{10}=\frac{10}{2}\left(2\times\frac{9}{4}+9\times\frac{3}{2}\right)$$

[1]

Evaluate.

$$S_{10}=5\left(\frac{18}{4}+\frac{27}{2}\right)$$

$$S_{10}=5\times 18$$

$S_{10}=90$ [1]

13b

If a is the first term and r is the common difference then the sum of the first two terms being 16 can be expressed as

$$a+ar=16$$

[1]

Using $S_\infty=\frac{a}{1-r}$, the sum to infinity is 25.

$$\frac{a}{1-r}=25$$

[1]

We can isolate a in both the above equations. Starting with $a + ar = 16$, factorise the left hand side.

$$a(1+r) = 16$$

Divide by $1+r$.

$$a = \frac{16}{1+r}$$

Turning our attention to $\frac{a}{1-r} = 25$, multiply by the denominator.

$$a = 25(1-r)$$

Now equate both the above.

$$\frac{16}{1+r} = 25(1-r)$$

Multiply by $1+r$.

$$16 = 25(1-r)(1+r)$$

[1]

Solve.

$$\frac{16}{25} = (1-r)(1+r)$$

$$\frac{16}{25} = 1 - r^2$$

$$r^2 = \frac{9}{25}$$

$$r = \pm\frac{3}{5}$$

[1]

$$r = \frac{3}{5},\ a = \frac{16}{1+\frac{3}{5}} = 16 \div \frac{8}{5} = 16 \times \frac{5}{8} = 2 \times 5$$

$a = 10$ [1]

$$r = -\frac{3}{5},\ a = \frac{16}{1-\frac{3}{5}} = 16 \div \frac{2}{5} = 16 \times \frac{5}{2} = 8 \times 5$$

$a = 40$ [1]

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Find an expression for the coefficient of the x^4 term of the expansion.

$$\frac{n!}{4!(n-4)!}\times(1)^{n-4}\times\left(\frac{1}{2}\right)^4$$

This can be written as

$$\frac{n(n-1)(n-2)(n-3)\times(n-4)!}{4!\times(n-4)!}\times\left(\frac{1}{2}\right)^4$$

Simplify.

$$\frac{n(n-1)(n-2)(n-3)}{1\times2\times3\times4}\times\left(\frac{1}{2}\right)^4$$

[1]

14

Similarly, find an expression for the x^6 coefficient of the term of the expansion.

$$\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{1\times2\times3\times4\times5\times6}\times\left(\frac{1}{2}\right)^6$$

[1]

We are told that the coefficient of x^4 is half the coefficient of x^6, therefore:

$$\frac{n(n-1)(n-2)(n-3)}{24\times16}=\frac{1}{2}\times\left(\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{720\times64}\right)$$

[1]

Simplify.

$$\frac{2\times720\times64}{24\times16}=\frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{n(n-1)(n-2)(n-3)}$$

$$\frac{720}{3}=(n-4)(n-5)$$

$$240=(n-4)(n-5)$$

[1]

Expand the brackets, rearrange and solve.

$$n^2 - 9n + 20 = 240$$
$$n^2 - 9n - 220 = 0$$
$$(n - 20)(n + 11) = 0$$

[1]

$n = 20$ or $n = -11$

The question asks for the positive value.

$n = 20$ [1]

15

i) Use binomial expansion to obtain the first three terms

$$(1 + 3x)^6 = 1^6 + {}^6C_1 \times 1^5 \times (3x)^1 + {}^6C_2 \times 1^4 \times (3x)^2 + \ldots$$

an unsimplified correct substitution or any two correct final terms from the answer below [1]

Simplify

$$= 1 + 6 \times 3x + 15 \times 9x^2$$

$1 + 18x + 135x^2$ [1]

ii) The expansion of $(a + x)^2$ is $a^2 + 2ax + x^2$. Multiply the first three terms of $(1 + 3x)^6$ found in (i) by $a^2 + 2ax + x^2$, up to terms in x^2 (you don't need to multiply terms that will result with x^3 or higher)

$$(1 + 18x + 135x^2 + \ldots)(a^2 + 2ax + x^2)$$
$$= (a^2)(1) + (a^2)(18x) + (a^2)(135x^2) + \ldots + (2ax)(1) + (2ax)(18x) + \ldots + (x^2)(1) + \ldots$$

(The line above is really just shown for demonstration purposes. In reality we'd probably go straight to the line below)

$$(1 + 18x + 135x^2 + \ldots)(a^2 + 2ax + x^2) = a^2 + 18a^2x + 135a^2x^2 + \ldots + 2ax + 36ax^2 + \ldots + x^2 + \ldots$$

We can equate this to $4 + 68x + bx^2$

$$a^2 + 18a^2x + 135a^2x^2 + \ldots + 2ax + 36ax^2 + \ldots + x^2 + \ldots = 4 + 68x + bx^2$$

The only constant term on the left hand side is "a^2" therefore equating the constants gives

$$a^2 = 4$$
$$a = \pm 2$$

[1]

Only one of these can be correct. Examining the terms in x :

$$a = -2,\ 18a^2x + 2ax = 18(-2)^2x + 2(-2)x = 68x$$
$$a = 2,\ 18a^2x + 2ax = 18(2)^2x + 2(2)x = 76x$$

But we know that the term in x is $68x$ therefore $a \neq 2$ and

$a = -2$ [1]

Now substitute $a = -2$ into the terms in x^2

$$135(-2)^2x^2 + 36(-2)x^2 + x^2 = bx^2$$
$$540x^2 - 72x^2 + x = bx^2$$
$$469x^2 = bx^2$$

Therefore

$b = 469$ [1]

16a

i) Imagine picking the first photograph. It must be of a sunset so there are 3 ways this can happen

$$3 \times \ldots$$

Ignore the middle 10 photographs for now; the last photograph must be of an ocean so there are 4 ways this can happen

$$3 \times \ldots \times 4$$

The middle 10 photographs are made up of the remaining 10 photographs. So there are 10 photographs for the first of these to be chosen from, 9 remaining for the next one to be chosen from, 8 remaining for the next one, and so on

$$3 \times 10! \times 4$$

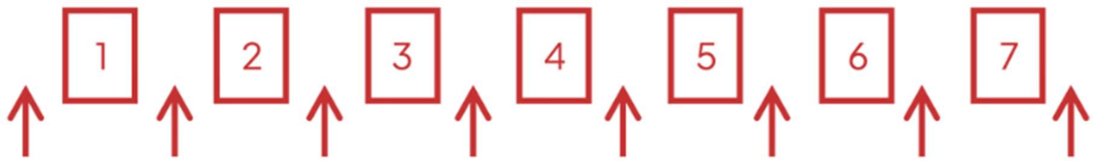
[1]

43 545 600 [1]

ii) First consider the five mountain photographs that can be placed next to each other. This can happen in 5! ways

$$5! \times \ldots$$

Think of the 5 mountain photographs as one 'block' of photographs. They could be placed in 8 different ways in between the 7 remaining photographs. The arrows diagram below illustrates the possible placements of the mountain 'block' around the 7 other photographs



$$5! \times 8 \times \ldots$$

Ignoring where between them the block of mountain photographs are placed, the 7 remaining photographs can be arranged in 7! ways

$$5! \times 8 \times 7!$$

[1]

4 838 400 [1]

16b

i) If no sunset photographs are included, only the 4 ocean photographs and 5 mountain photographs are to be chosen from, so 9 in total. And as it doesn't matter whether they are ocean or mountain, we're just choosing 3 photographs from 9. Hence

$${}^{9}C_{3}$$

[1]

Evaluating this gives

84 [1]

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ii) One photograph is to be chosen from the 3 sunsets, so there are 3 ways to do this. For each of these 3 ways, there are 4 ways to choose an ocean photograph giving

$$3 \times 4$$

And for each of these 12 ways of choosing 1 sunset and one ocean, there are another 5 ways to choose one from the mountain photographs. So the answer can be calculated by

$$3 \times 4 \times 5$$

[1] *note that this is the same as* ${}^3C_1 \times {}^4C_1 \times {}^5C_1$

Evaluate

60 [1]

17

i) There are 6 digits and 5 spaces and order does not matter

$$6P5$$

or

$$6 \times 5 \times 4 \times 3 \times 2$$

720 [1]

ii) Find how many numbers are divisible by 5

Since we have no 0, it is just numbers that end in 5

So we now have 5 choices for the first digit, 4 for the second, 3 for the third, 2 for the fourth and only 1 for the fifth (as it has to be a 5)

$$5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$(\text{or } 5P4 = 120)$$

Subtract the numbers that are divisible by 5 from part i)

$$720 - 120 = 600$$

600 [1]

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iii) Numbers that are even will end in 2 or 8 (from the numbers we are given)

Numbers that are greater than 30,000 will start with a 3, 5, 7 or 8

plan for adding numbers ending in 2 and numbers ending in 8 [1]

Case 1: ends in 2, starts in 3, 5, 7 or 8

There are 4 choices for the first digit, 1 choice for the last digit, 4 choices left for the second digit, 3 choices for the third digit and 2 choices for the fourth digit

$$4 \times 4 \times 3 \times 2 \times 1 = 96$$

[1]

Case 2: ends in 8, starts in 3, 5, or 7

There are 3 choices for the first digit, 1 choice for the last digit, 4 choices left for the second digit, 3 choices for the third digit and 2 choices for the fourth digit

$$3 \times 4 \times 3 \times 2 \times 1 = 72$$

[1]

Work out the total

$$96 + 72 = 168$$

168 [1]

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18

use of Binomial expansion to find a term in either $\left(\frac{1}{3x^2}-\frac{x}{2}\right)^9$, $\left(\frac{1}{3x^{7/3}}-\frac{x^{2/3}}{2}\right)^9$,

$\left(\frac{1}{3}-\frac{x^3}{2}\right)^9$, $\left(\frac{1}{3x^3}-\frac{1}{2}\right)^9$ or $\left(2-3x^3\right)^9$ ***(M1)(A1)***

Note: Award ***M1*** for a product of three terms including a binomial coefficient and powers of the two terms, and ***A1*** for a correct expression of a term in the expansion.

finding the powers required to be 2 and 7 ***(M1)(A1)***

constant term is ${}^9C_2\times\left(\frac{1}{3}\right)^2\times\left(-\frac{1}{2}\right)^7$ ***(M1)***

Note: Ignore all x's in student's expression.

therefore term independent of x is $-\frac{1}{32}$ $(=-0.03125)$ ***A1***

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19

(a) $2x^2 + x - 3 = (2x+3)(x-1)$

Note: Accept $2\left(x+\frac{3}{2}\right)(x-1)$.

Note: Either of these may be seen in (b) and if so ***A1*** should be awarded.

(b) **EITHER**

$$(2x^2 + x - 3)^8 = (2x+3)^8(x-1)^8$$
$$= \left(3^8 + 8(3^7)(2x) + \ldots\right)\left((-1)^8 + 8(-1)^7(x) + \ldots\right)$$

coefficient of $x = 3^8 \times 8 \times (-1)^7 + 3^7 \times 8 \times 2 \times (-1)^8$
$= -17\,496$

Note: Under ft, final ***A1*** can only be achieved for an integer answer.

OR

$$(2x^2 + x - 3)^8 = \left(3 - (x - 2x^2)\right)^8$$
$$= 3^8 + 8\left(-(x-2x^2)(3^7) + \ldots\right)$$

coefficient of $x = 8 \times (-1) \times 3^7$
$= -17\,496$