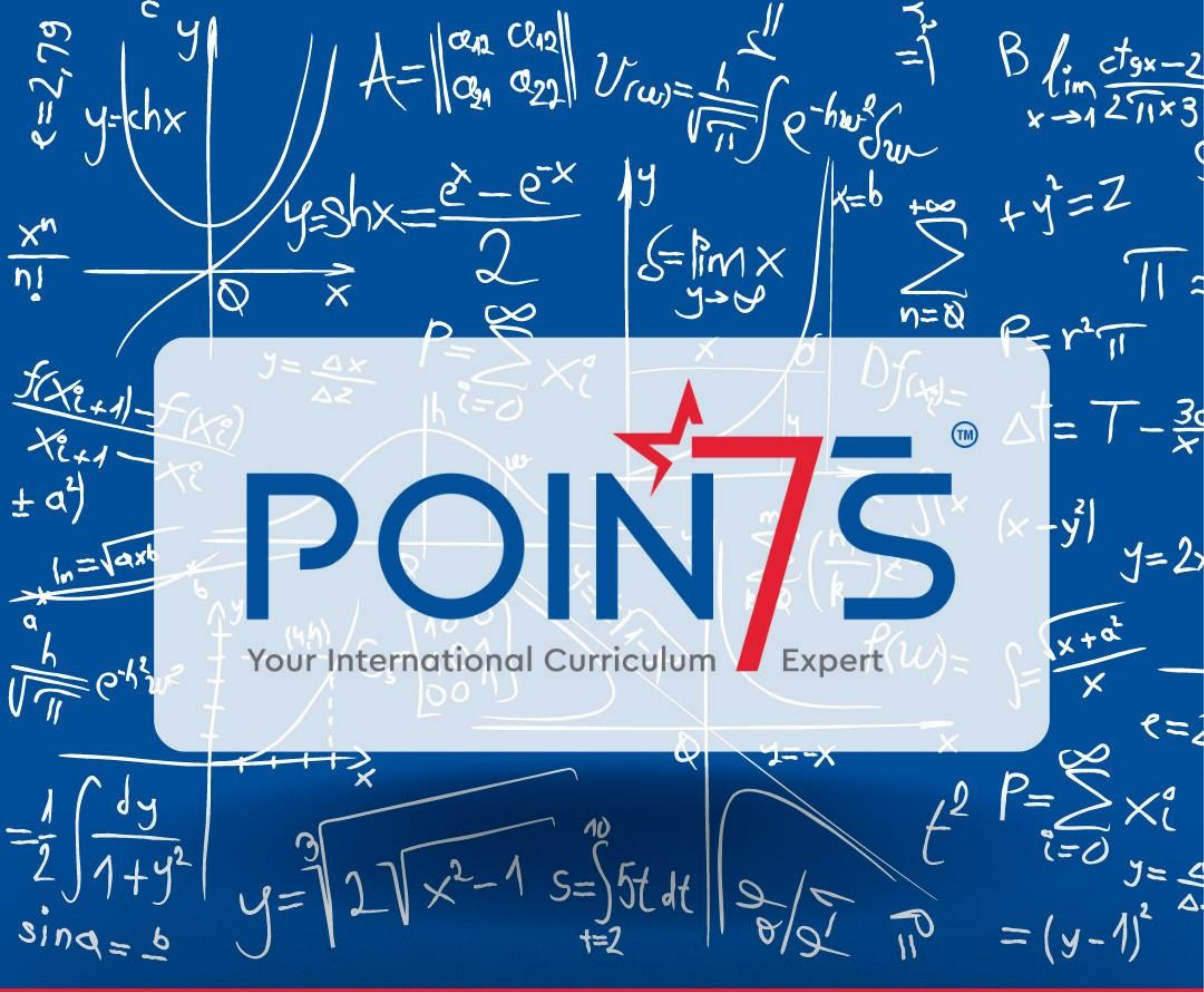

POINTS

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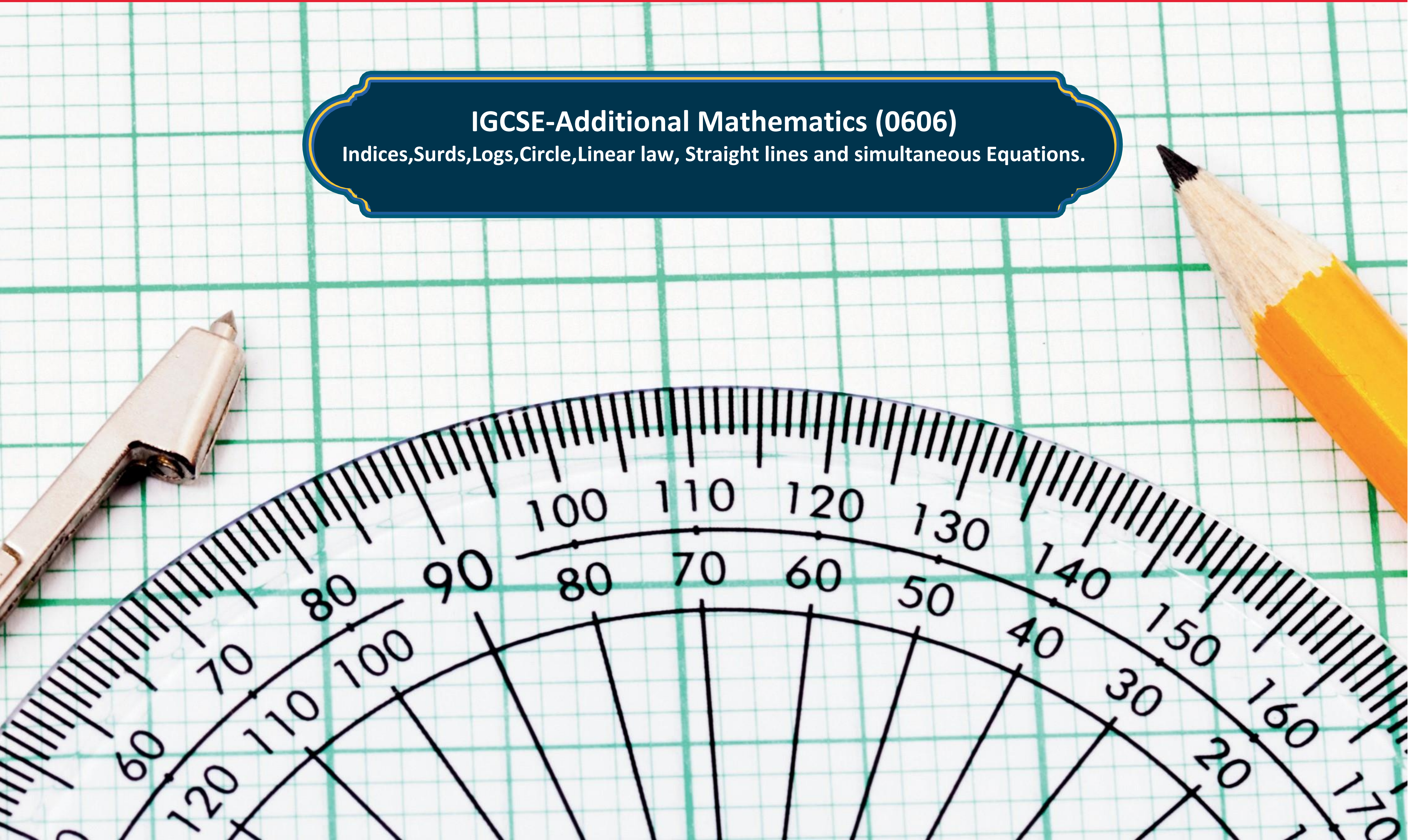
# ELEVATE

## MATH TOPICAL WORKSHEETS

**IGCSE-Additional Mathematics (0606)**

Indices,Surds,Logs,Circle,Linear law, Straight lines and simultaneous Equations.



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|---|---|
| **Topic:** | Indices,Surds,Logs,Circle,Linear law, Straight lines and simultaneous Equations. |
| **Board:** | IGCSE |
| **Subject:** | Additional Mathematics (0606) |

1

| | | | |
|---|---|---|---|
| | $m_{AB} = \frac{-9-3}{6--4}$ oe isw or $-\frac{6}{5}$ | M1 | |
| | $[m_\perp =]\ \frac{5}{6}$ or $\frac{-1}{their\ m_{AB}}$ | M1 | FT *their* $m_{AB}$ providing that $\frac{\text{difference in } y}{\text{difference in } x}$ attempted |
| | [Midpoint] $\left(\frac{-4+6}{2}, \frac{3-9}{2}\right)$ oe isw or $(1, -3)$ | M1 | |
| | Finds equation of perpendicular bisector: $y - their\ (-3) = -\frac{1}{their\left(-\frac{6}{5}\right)}(x - their\ 1)$ oe | M1 | FT *their* perpendicular gradient providing gradient is $\frac{-1}{their\ m_{AB}}$ and using *their* midpoint<br>*Their* midpoint must not be any of $A$, $B$, $C$ or $D$ |
| | $11x - 75 = \frac{5}{6}x - \frac{23}{6}$ | M1 | Eliminates one variable<br>**dep** on previous **M** mark<br>$11x - 75 =$ *their* perp bisector equation |
| | $E(7, 2)$ | A1 | |
| | [Area $CDE$ =] 40 | A1 | Dependent on all previous marks |

2

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|---|---|---|
| $5x^2 - 8x - 4\ \ (=0)$ or $5y^2 - 36y - 305\ \ (=0)$ | M1 | For attempt to eliminate one variable to obtain a 3 term quadratic equation (= 0). Allow one sign error. |
| $x = -\frac{2}{5},\ x = 2$<br>$y = -\frac{61}{5},\ y = -5$ | 3 | **Dep M1** for attempt to solve *their* quadratic equation<br>**A1** for any correct pair<br>**A1** for a second correct pair. |
| Mid-point $\left(\frac{4}{5},\ -\frac{43}{5}\right)$ | M1 | For attempt to find mid-point using *their* coordinates |
| Gradient of perpendicular $= -\frac{1}{3}$ | B1 | |
| $y + \frac{43}{5} = -\frac{1}{3}\left(x - \frac{4}{5}\right)$ | M1 | For attempt to find the equation of the perpendicular bisector using *their* perpendicular gradient and *their* midpoint. Allow alternative methods |
| $a = -1$ | A1 | |



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3

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|---|---|---|---|
| (a) | $\ln y = mx^3 + c$ soi | **B1** | $c$ may be shown as a log term |
| | $25 = -8m + c$<br>$5 = 2m + c$ | **M1** | For at least one correct equation, may be used with gradient of $-2$ |
| | $m = -2,\ c = 9$ | **A1** | $y = -2x + 9$ seen implies **M1A1** |
| | $y = e^{9-2x^3}$ oe | **A1** | |
| (b) | $25 = 9 - 2x^3$ | **M1** | For equating exponential indices and obtaining $x = \ldots$<br>or use of $x = \sqrt[3]{\frac{25 - their\ 9}{their - 2}}$ |
| | $x = -2$ only | **A1** | |

4

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|---|---|---|---|
| (a) | [Gradient =] $\frac{15.4-10.4}{4-2}$ oe soi | M1 | |
| | $10.4 = their2.5 \times 2 + c$ or $15.4 = their2.5 \times 4 + c$<br>or<br>$\frac{y-10.4}{x-2} = their2.5$ or $\frac{y-15.4}{x-4} = their2.5$ | M1 | FT *their* gradient |
| | [Gradient = ] 2.5 soi and [intercept =] 5.4 soi | A1 | |
| | $\sqrt{y} = 2.5\log_2(x+1)+5.4$ oe isw | A1 | |
| | **Alternative method** | | |
| | $10.4 = 2m + c$ and $15.4 = 4m + c$<br>and solving to find $m$ or $c$ | (M1) | |
| | Use *their* $m$ or $c$ to find *their* $c$ or $m$ | (M1) | |
| | $m = 2.5$ and $c = 5.4$ | (A1) | |
| | $\sqrt{y} = 2.5\log_2(x+1)+5.4$ oe isw | (A1) | |
| (b) | $\frac{5929}{25}$ or 237.16 | B1 | |
| (c) | $5 = their2.5\log_2(x+1) + their5.4$<br>and rearrange to make $\log_2(x+1)$ the subject | M1 | **FT** *their* equation from **(a)** of correct form with $m \neq 1$ or 0, and $c \neq 0$<br><br>Condone any base |
| | $-\frac{4}{25} = \log_2(x+1)$ oe | A1 | Condone any base |
| | $x = -0.105$ or $-0.1049[74\ldots]$ rot to 4 or more sf | A1 | |

5

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|---|---|---|---|
| (a) | $a=\frac{10}{3}$ or $3\frac{1}{3}$ | **B1** | |
| | $b=\frac{7}{3}$ or $2\frac{1}{3}$ | **B1** | |
| | $c=\frac{9}{2}$ or $4\frac{1}{2}$ or $4.5$ | **B1** | |
| (b) | $10\left(2^p\right)^2-17\left(2^p\right)+3=0$<br>$\left(5\left(2^p\right)-1\right)\left(2\left(2^p\right)-3\right)=0$<br>$2^p=\frac{1}{5},\ \ 2^p=\frac{3}{2}$ | **M1** | For recognition of a quadratic in $2^p$, attempt to factorise and solve for $2^p$ |
| | $p=\frac{\ln\frac{1}{5}}{\ln 2}$ or $p=\frac{\ln 1.5}{\ln 2}$ oe | **M1** | For correct attempt to deal with $2^p=k$ |
| | $-2.32$ | **A1** | |
| | $0.585$ | **A1** | |

6

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|---|---|---|---|
| | Eliminates one unknown<br>e.g. $\frac{x^2}{4}+\frac{1}{9}\left(\frac{3}{2x}\right)^2=1$ | **M1** | |
| | Rearranges to solvable form<br>e.g. $x^4-4x^2+1=0$ | **A1** | |
| | Solves :<br>$\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2}$ | **M1** | **dep** on attempt to eliminate one unknown and having a 3-term quadratic in $x^2$ |
| | $x^2=2\pm\sqrt{3}$ oe isw<br>or 3.7320[5...] and 0.2679[4...] | **A1** | |
| | $x=\pm\,1.932$ or $x=\pm\,0.518$ | **A1** | |

7

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| | Finds by elimination $3y+\sqrt{7}y=4$ oe<br>or substitutes $x=11-3y$ into $x-\sqrt{7}y=7$ oe<br><br>OR<br>Finds by elimination $3y+\sqrt{7}y=21+11\sqrt{7}$ oe<br>or substitutes $y=\frac{11-x}{3}$ into $x-\sqrt{7}y=7$ oe | M1 | |
| | $y=\frac{4}{3+\sqrt{7}}$<br><br>or $x=\frac{21+11\sqrt{7}}{3+\sqrt{7}}$ | A1 | |
| | $y=\frac{4}{3+\sqrt{7}}\times\frac{3-\sqrt{7}}{3-\sqrt{7}}$ oe<br><br>or $x=\frac{21+11\sqrt{7}}{3+\sqrt{7}}\times\frac{3-\sqrt{7}}{3-\sqrt{7}}$ oe | M1 | **FT** *their* value of $x$ or $y$ providing of equivalent difficulty |
| | $y=6-2\sqrt{7}$ and $x=6\sqrt{7}-7$ | A2 | **A1** for either and no extra values |

8.

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| (a) | $(3-4)^2+(-1+3)^2=5$<br>or showing that distance between point $A$ and centre of circle = radius e.g.<br>$\sqrt{(3-4)^2+(-1+3)^2}=\sqrt{5}$ | B1 | Accept if $x$-coordinate substituted to find $y$- coordinate |
| (b) | $\begin{pmatrix}4\\-3\end{pmatrix}+\begin{pmatrix}1\\-2\end{pmatrix}$ oe, soi<br><br>or using centre and point $A$ e.g.<br>$\frac{3+x}{2}=4$ and $\frac{-1+y}{2}=-3$<br>or solve simultaneously the line $y=-2x+5$ with the circle leading to 3 term quadratic $x^2-8x+15[=0]$ with an attempt to solve | M1 | Award **M1** for the 2 equations<br><br>Award the **M1** for the quadratic |
| | (5, –5) | A1 | |

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| (c) | gradient of radius $= \frac{-3+1}{4-3} = -2$ soi by gradient of tangent | M1 | |
| | gradient of tangent $= \frac{-1}{their(-2)}$ | M1 | FT *their* –2<br>Allow if differentiation is used e.g.: |
| | $y + 1 = \frac{1}{2}(x - 3)$ oe | A1 | FT $\frac{-1}{their-2}$<br>ISW from a correct unsimplified answer |
| | **Alternative** | | |
| | use of differentiation e.g.:<br>$(y+3)^2 = 5-(x-4)^2$<br>$y = \sqrt{-x^2+8x-11}-3$<br>$\frac{dy}{dx} = \frac{-2x+8}{2\sqrt{-x^2+8x-11}}$ | (M1) | allow one error<br>or use of implicit differentiation (not on syllabus) e.g.:<br>$2x+2y\frac{dy}{dx}-8+6\frac{dy}{dx}=0$<br>leading to $\frac{dy}{dx}=\frac{8-2x}{2y+6}$ |
| | Substitute $(3,-1)$ in *their* $\frac{dy}{dx}$<br>to get gradient $=\frac{1}{2}$ | (M1) | **Dep** on first **M1** |
| | $y + 1 = \frac{1}{2}(x - 3)$ oe | (A1) | ISW from a correct unsimplified answer |

**9.**

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| (a) | Centre $(5,\ 12)$ soi | B1 | |
| | Gradient of radius $= \frac{12-4}{5-11}$ $\left(=-\frac{4}{3}\right)$ | M1 | |
| | Equation of normal: $y-4=\frac{3}{4}(x-11)$ | M1 | Must be using *their* perpendicular gradient and the correct point |
| | $3x-4y=17$ | A1 | |
| (b) | Distance between centres = 13<br>Sum of radii = 12 | M1 | For both |
| | $13>12$ so no intersection | A1 | |

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| **10.** (a) | $4(y-3)^2 = 36-9$ or $4y^2 - 24y + 9 [= 0]$ | **M1** | |
| | $y = 3 \pm \sqrt{\frac{27}{4}}$ or exact equivalent, soi | **A1** | |
| | $3 + \sqrt{\frac{27}{4}} - \left(3 - \sqrt{\frac{27}{4}}\right)$ oe | **M1** | **FT** *their* $a \pm \sqrt{b}$ providing $b$ is not a square number |
| | $\sqrt{27}$ or $3\sqrt{3}$ or exact equivalent, nfww | **A1** | |
| (b) | Eliminates one unknown and simplifies terms: $2x^2 + 83 = x^2 - 20x$ oe, soi | **M1** | |
| | $x^2 + 20x + 83 = 0$ | **A1** | |
| | Applies quadratic formula or completes the square: $x = \dfrac{-20 \pm \sqrt{20^2 - 4[1](83)}}{2}$ | **M1** | **FT** *their* 3-term quadratic |
| | $x = -10 \pm \sqrt{17}$ | **A1** | |
| | $y = \dfrac{1}{-10 \pm \sqrt{17}} \times \dfrac{-10 \mp \sqrt{17}}{-10 \mp \sqrt{17}}$ | **M1** | **FT** *their* $x = a \pm \sqrt{b}$ providing previous **M1** awarded |
| | $x = -10 \pm \sqrt{17}$ , $y = -\dfrac{10}{83} \mp \dfrac{\sqrt{17}}{83}$ | **A1** | **dep** on all marks previously awarded |

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| **11.** (a) | Forms a correct equation from which the logarithms can be eliminated :<br><br>has consistent powers of ln*x* in all terms<br>$\frac{n}{2}\left(12\ln x+(n-1)\times4\ln x\right)*43\times24\ln x$<br>oe, soi<br><br>OR<br><br>uses log laws to combine terms<br>$\ln\left(x^{6n}\times x^{2n(n-1)}\right)*\ln x^{24\times43}$ or better | 3 | where * is = or any inequality sign<br><br>**M2** for $a = 6$ **and** $d = 4$<br>or<br>$a = 6\ln x$ soi **and** $d = 4\ln x$ soi<br>or<br>$a = \ln x^6$ soi **and** $d = \frac{2}{3}\ln x^6$<br>soi<br>or<br>$a = \frac{3}{2}\ln x^4$ soi **and** $d = \ln x^4$<br>soi<br><br>or for<br>$\ln x^{12\times\frac{n}{2}}+\ln x^{4(n-1)\times\frac{n}{2}}*\ln x^{24\times43}$<br>oe<br>or<br>**M1** for any correct expression for *d*<br>or<br>for a correct expression for the sum to *n* terms using *their a* and *their d* providing *their d* is not one of the given terms |
| | $2n^2+4n-1032[*0]$ oe | A1 | |
| | Solves *their* 3-term quadratic in *n* | M1 | **dep** on attempt at the sum of an AP |
| | $n = 22$ cao, nfww | A1 | **dep** on all previous marks awarded |

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| **12.** (a) | Solves or factorises $x^{\frac{1}{3}} - x^{\frac{1}{6}} - 2 = 0$ oe: $\left(x^{\frac{1}{6}}+1\right)\left(x^{\frac{1}{6}}-2\right)$ or when substituting $y = x^{\frac{1}{3}}$ factorises to $\left(y^{\frac{1}{2}}+1\right)\left(y^{\frac{1}{2}}-2\right)$ | **M1** | A substitution may be used $\left(x^{\frac{1}{3}}\right)^6 - \left(x^{\frac{1}{6}}\right)^6 = 2^6$ scores 0 marks $\left(x^{\frac{1}{3}}\right) - \left(x^{\frac{1}{6}}\right) = x^{\log_x 2}$ then $\frac{1}{3} - \frac{1}{6} = \log_x 2$ scores 0 marks |
| | $\left[x^{\frac{1}{6}} = -1\right]$, $x^{\frac{1}{6}} = 2$ | **A1** | |
| | $x = 64$ [$x = 1$] | **A1** | |
| | Rejects $x^{\frac{1}{6}} = -1$ or $x = 1$ ignored at some stage, soi | **B1** | |
| (b) | $x + 2y = 1$ | **B1** | |
| | Correctly eliminates one unknown | **M1** | **Dep** on **B1** allow unsimplified e.g. $x = 1 - 2y$ or $y = \frac{1-x^2}{4x+1}$ |
| | Correct quadratic in solvable form: $y - 4y^2 [= 0]$ oe or $2x^2 - 3x + 1 [= 0]$ oe | **A1** | |
| | Factorises or solves *their* quadratic to get two solutions | **M1** | **Dep** on previous **M1** |
| | $y = 0$, $x = 1$ and $y = \frac{1}{4}$, $x = \frac{1}{2}$ | **A1** | |

**13.**

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|---|---|---|---|
| | $6\left(2^{2x}\right) - 11\left(2^x\right) + 3 = 0$ soi | **B1** | |
| | $2^x = \frac{1}{3}$, $2^x = \frac{3}{2}$ | **2** | **M1** for solution of *their* quadratic equation in $2^x$ or appropriate substitution to obtain $2^x = \ldots$ |
| | $-1.58$, $0.58$ | **A1** | For both |

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| **14.** | (a) | $(3x + 1)\log 2 = (x - 2)\log 5$ oe | **B1** | |
| | | $(3\log 2 - \log 5)x = -\log 2 - 2\log 5$ | **M1** | **FT** if of equivalent difficulty |
| | | $x = -8.32$ | **A1** | |
| | (b) | Writes as a quadratic in $e^{2y+1}$ or states $u = e^{2y+1}$ and writes as a quadratic in $u$ oe, soi | **M1** | condone one error |
| | | $\left(e^{2y+1}\right)^2 - e^{2y+1} - 6 \quad [=0]$ oe<br>or $u^2 - u - 6 \quad [=0]$ oe | **A1** | |
| | | $(e^{2y+1} + 2)(e^{2y+1} - 3) \quad [=0]$ leading to $e^{2y+1} = 3$<br>or $(u + 2)(u - 3)$ $[=0]$ leading to $e^{2y+1} = 3$ | **A1** | |
| | | $y = 0.0493$ and no other solutions | **A1** | |