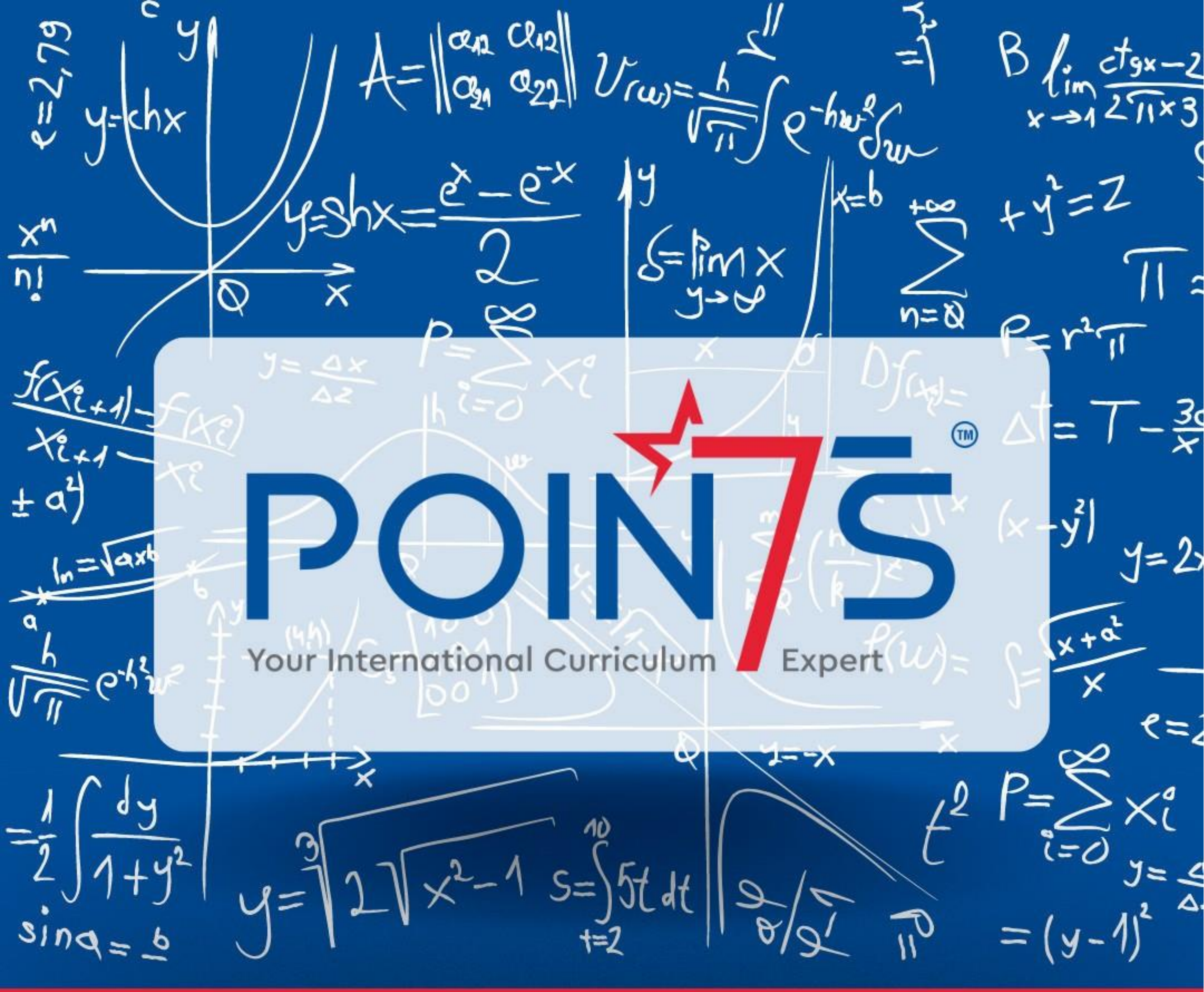
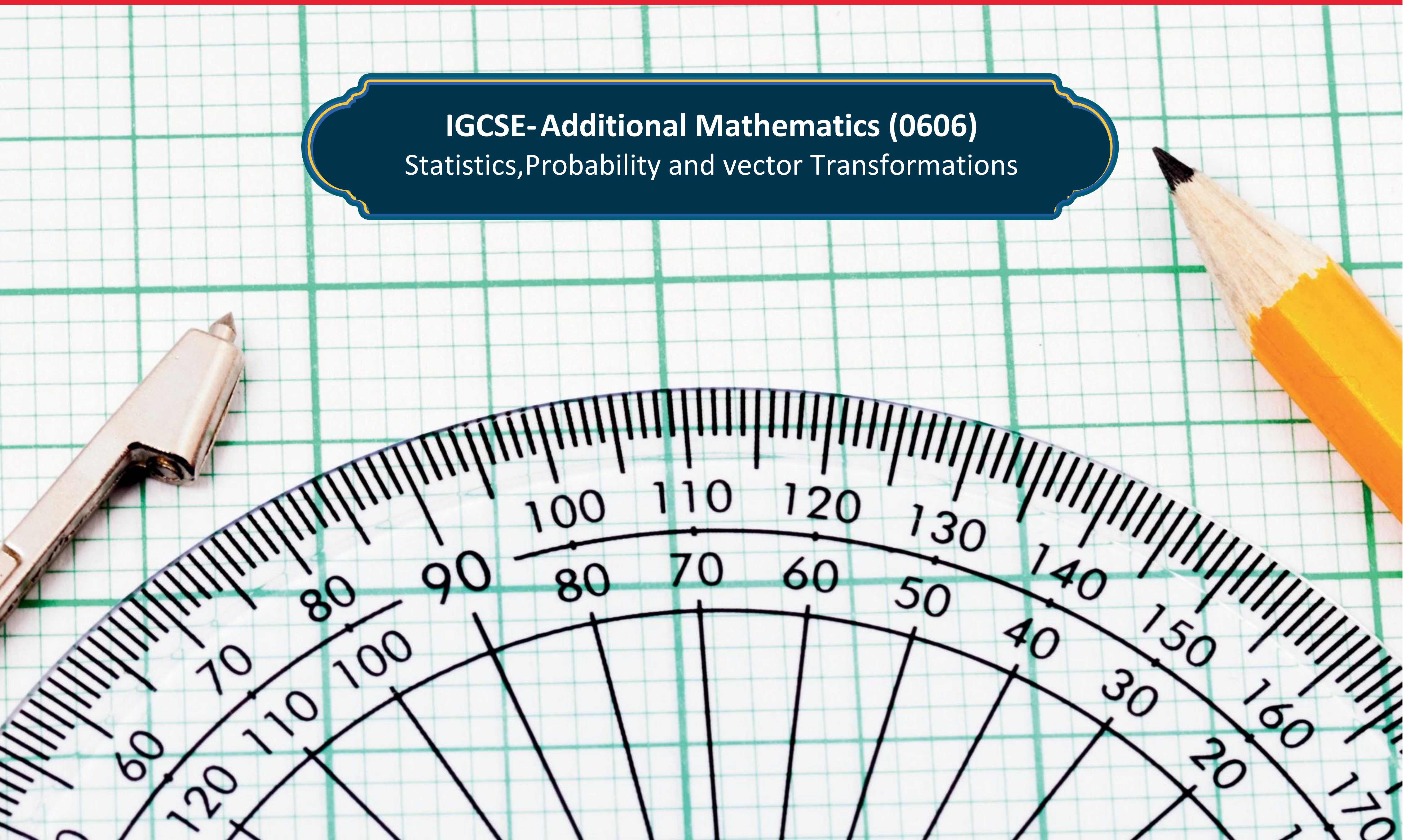


POINTS

Your International Curriculum Expert

ELEVATE

MATH TOPICAL WORKSHEETS



IGCSE- Additional Mathematics (0606)

Statistics,Probability and vector Transformations

1

Find the total number of balls in a bag.

$$10 + 8 = 18$$

Write the number of red balls as a fraction of the total number of balls.

$$\frac{10}{18}$$

Simplify your answer.

$\frac{5}{9}$ [1]

2

i) Find the probability that the first ball picked at random will be red, given that there are 6 balls and 4 of them are red.

$$\frac{4}{6} = \frac{2}{3}$$

Find the probability that the second ball will be red, given that there are now 5 balls and 3 of them are red as the first red ball was not replaced.

$$\frac{3}{5}$$

Multiply the probabilities together to find the probability that two red balls will be picked.

$$\frac{2}{3} \times \frac{3}{5}$$

[1]

$\frac{2}{5}$ [1]

Any equivalent fractional, decimal or percentage answer will be accepted.

+918591171615 https://pointsedulab.com/ info@pointsedulab.com

ii) Subtract the probability that the marbles are both red (from part i)) from 1 to find the probability that the marbles picked at random are NOT both red.

$$1 - \frac{2}{5}$$

$\frac{3}{5}$ [1]

Any equivalent fractional, decimal or percentage answer will be accepted.

3

i) The first disc is not replaced before the second disc is chosen, so the events are conditional and the probabilities will change.

There are 2 discs with the number 3, if the first disc chosen has the number 3 then there will only be one disc left with the number 3 on it.

$$\text{P(Both number 3)} = \left(\frac{2}{6} \times \frac{1}{5}\right) = \frac{2}{30}$$

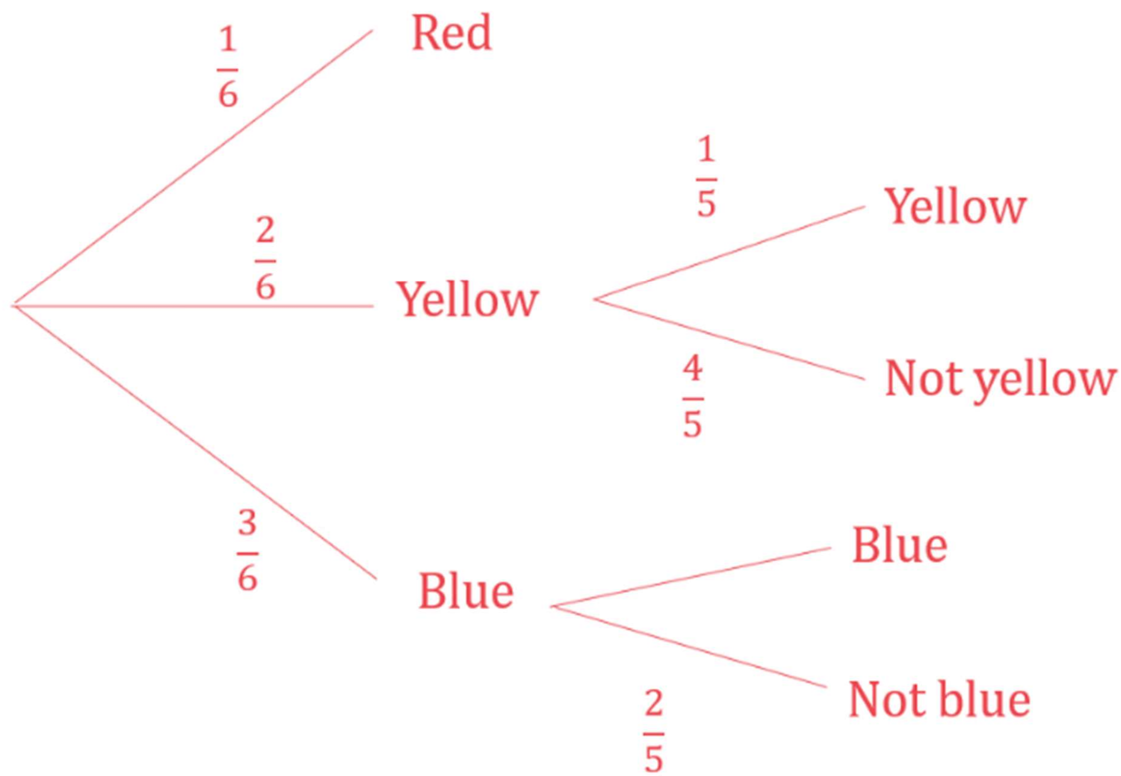
Correct method [1]

$\frac{1}{15}$ [1]

Unsimplified fraction or equivalent correct decimal accepted.

ii) The first disc is not replaced before the second disc is chosen, so the events are conditional and the probabilities will change.

This is best represented using a probability tree diagram, as there is only one red disc it is not possible to choose two red discs.



Multiply the probabilities with the same colour together and add the products.

$$\left(\frac{1}{6}\times 0\right)+\left(\frac{2}{6}\times\frac{1}{5}\right)+\left(\frac{3}{6}\times\frac{2}{5}\right)$$

1 mark for either of the non-zero probabilities [2]

$$\frac{0}{30}+\frac{2}{30}+\frac{6}{30}=\frac{8}{30}$$

$\frac{4}{15}$ [1]

Unsimplified fraction or equivalent correct decimal accepted.

4

Find the midpoint of each interval by adding together the endpoints then halving.

Mass (x tonnes)	$100 < x \leq 200$	$200 < x \leq 250$	$250 < x \leq 300$	$300 < x \leq 500$
Midpoint	150	225	275	400

[1]

Assume each value in any interval is its midpoint.
Multiply the midpoint by the frequency to find the total mass for that interval.

Mass (x tonnes)	150	225	275	400
Frequency	8	20	12	12
Total mass	1200	4500	3300	4800

Find the total mass by adding together the total mass for each interval.

$$1200 + 4500 + 3300 + 4800 = 13\,800$$

[1]

Estimate the mean by dividing the total mass by the total frequency (52 weeks).

$$13\,800 \div 52 = 265.3846\ldots$$

[1]

Round to 3 significant figures.

Estimate of the mean is 265 tonnes [1]

+918591171615 https://pointsedulab.com/ info@pointsedulab.com

5

The mode is the most occurring number, so $a = 7$

$$7, 7, b, c$$

[1]

There are four values, so the median, 8.5, is the midpoint between the 2nd value, 7, and the 3rd value, b $7 + 1.5 = 8.5$, so $8.5 + 1.5 = 10$

$$7, 7, 10, c$$

[1]

The mean of the four numbers is 9, so the sum of the four numbers divided by 4 equals 9

$$\frac{7+7+10+c}{4} = 9$$

[1]

Multiply both sides by 4 and use that $7 + 7 + 10 = 24$

$$7+7+10+c = 36$$
$$24 + c = 36$$

Solve the equation to find c (by subtracting 24 from both sides)

$$c = 36 - 24$$

$a = 7, b = 10, c = 12$ [1]

+918591171615 https://pointsedulab.com/ info@pointsedulab.com

6

The **area** of a bar on a histogram is **proportional** the frequency of that group.
Find the area of the bar for the group $0 < m \leq 4$ by multiplying its width (4 - 0) by its height.

$$4 \times 1.6 = 6.4$$

Find the scale factor between the area and the frequency by dividing the given frequency by the given area.

$$32 \div 6.4 = 5$$

[1]

Therefore we need to multiply the area of a bar by 5 to get the frequency.

Mass (m kg)	$0 < m \leq 4$	$4 < m \leq 6$	$6 < m \leq 7$	$7 < m \leq 10$
Frequency	32	Area = $(6-4) \times 2 = 4$ Freq = $4 \times 5 =$ **20**	Area = $(7-6) \times 1.2 = 1.2$ Freq = $1.2 \times 5 =$ **6**	42
Height of bar on histogram (cm)	1.6	2	1.2	2.8

[1]

+918591171615 https://pointsedulab.com/ info@pointsedulab.com

7

Find how many degrees women have in their sector

$$360° - 110° = 250°$$

[1]

Find how many "more" degrees women have in their sector compared to men

$$250° - 110° = 140° \text{ more}$$

These extra 140° represent the 3360 more women than men Find out how much 1° represents (by dividing 3360 by 140)

$$140° \text{ is } 3360 \text{ people so } 1° \text{ is } \frac{3360}{140} = 24 \text{ people}$$

[1]

Use the conversion 1° = 24 people to work out the total number of voters (by multiplying 360 by 24)

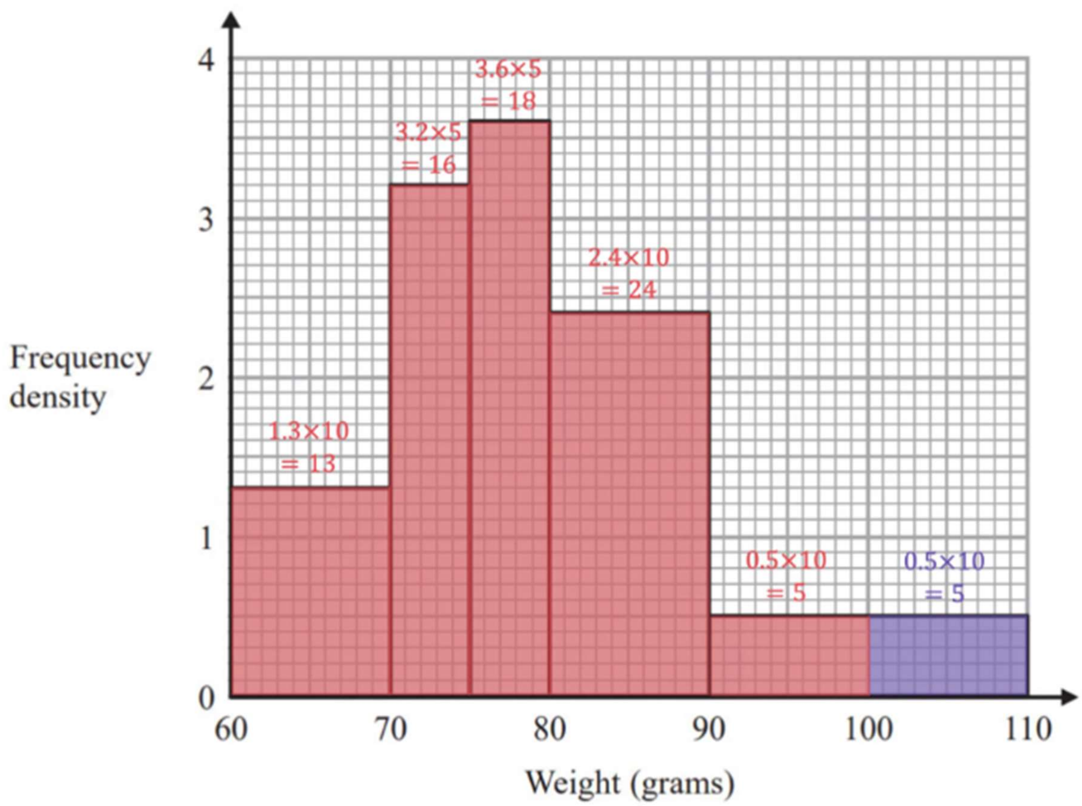
$$360 \times 24$$

8640 voters [1]

+918591171615 https://pointsedulab.com/ info@pointsedulab.com

8

Use Frequency = Frequency density × Class width to work out the frequency of each bar on the histogram.
The last bar will need splitting at a weight of 100 grams.



Work out the number of plums with a weight less than 100 grams, and the total number of plums.

<100: $13 + 16 + 18 + 24 + 5 = 76$

Total: $76 + 5 = 81$

One mark for each [2]

Now write the proportion as a fraction.

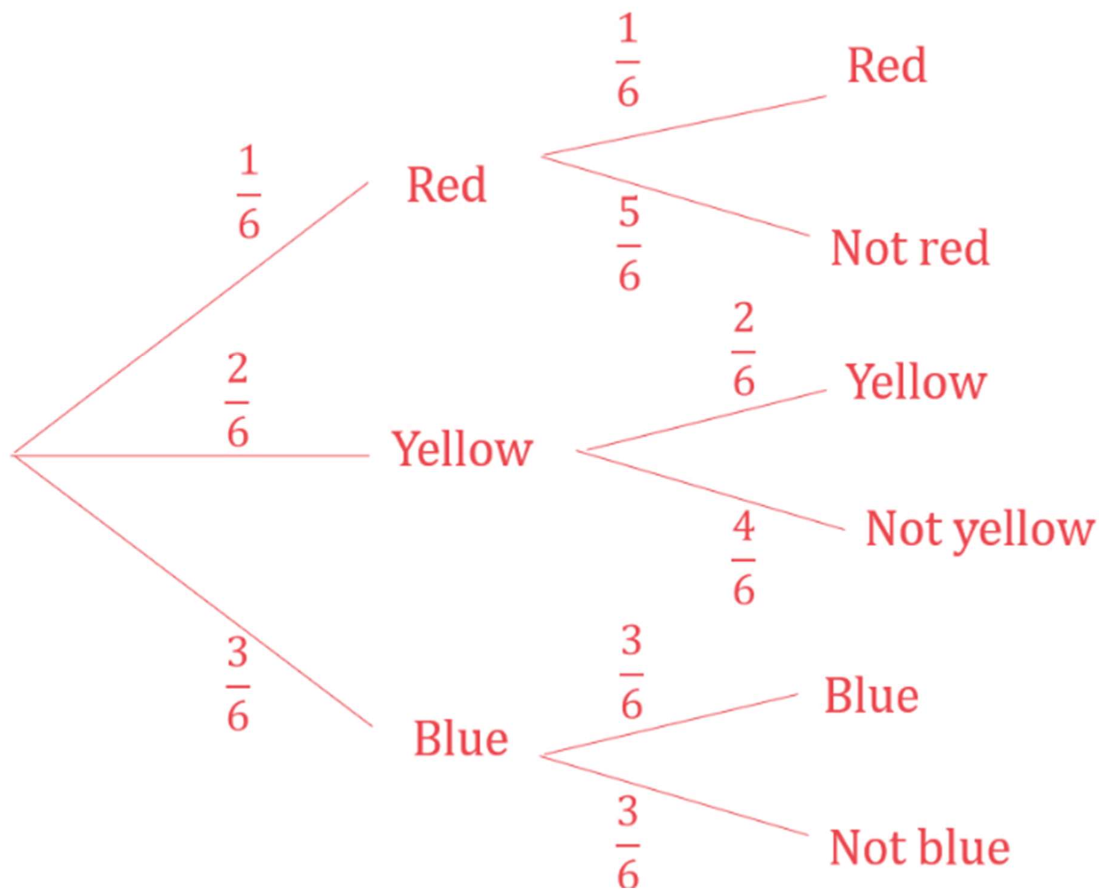
The proportion of plums with a weight less than 100 grams is $\frac{76}{81}$ [1]

A decimal or percentage answer is also allowed (0.938 or 93.8% (3 s.f.)).

9

The first disc is replaced before the second disc is chosen. The probabilities will not change and it is possible for both discs to be any of the colours.

Draw a probability tree, keeping the probabilities the same for the second disc.



+918591171615 https://pointsedulab.com/ info@pointsedulab.com

Multiply the probabilities with the same colour together and add the products.

$$\left(\frac{1}{6}\times\frac{1}{6}\right)+\left(\frac{2}{6}\times\frac{2}{6}\right)+\left(\frac{3}{6}\times\frac{3}{6}\right)$$

At least one of the probabilities correct [1]

Correct method [1]

Use your calculator to find the solutions.

$$\left(\frac{1}{6}\right)^2+\left(\frac{2}{6}\right)^2+\left(\frac{3}{6}\right)^2=\frac{14}{36}$$

$\frac{7}{18}$ [1]

Unsimplified fraction or equivalent correct decimal accepted.

10a

Cumulative frequency is the "running total" or the total of the frequencies up to the upper boundary of each class interval.

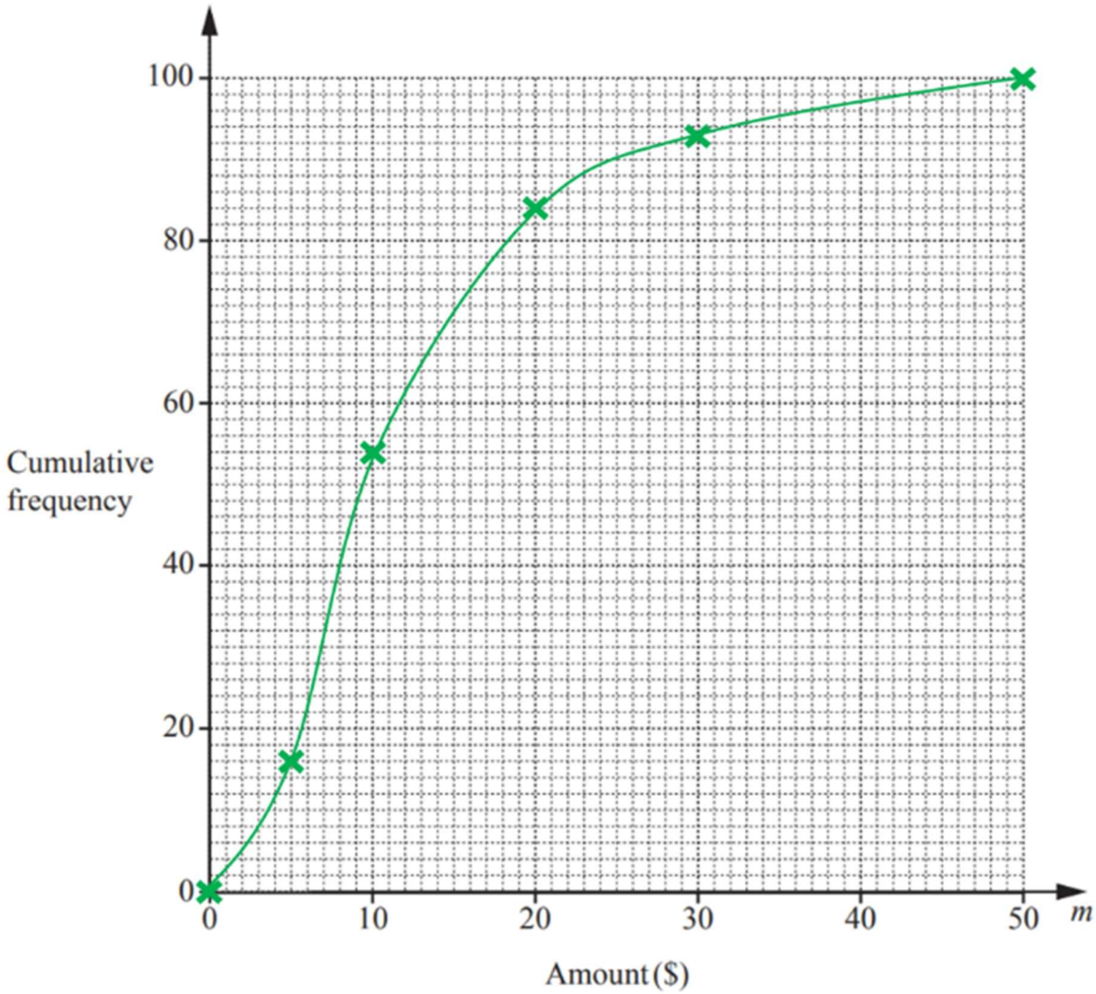
Amount ($\$ m$)	$m \le 5$	$m \le 10$	$m \le 20$	$m \le 30$	$m \le 50$
Cumulative Frequency	16	$16+38=$ **54**	$54+30=$ **84**	$84+9=$ **93**	100

1 mark for 2 correct, 2 marks for all correct [2]

+918591171615 https://pointsedulab.com/ info@pointsedulab.com

10b

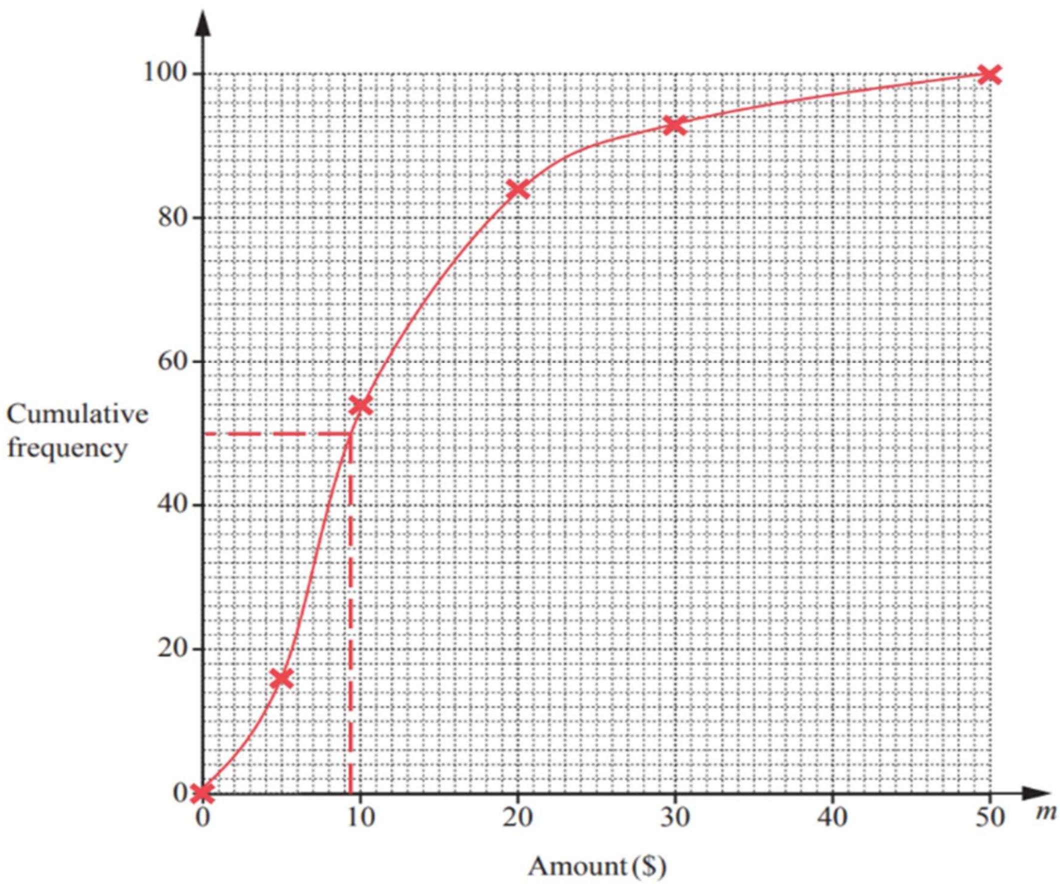
When plotting a cumulative frequency diagram, plot the points at the upper limit of the class interval.
(It is only by the upper limit that we have definitely included all the data in that class interval.)
Include a starting point at (0, 0), then plot points using the table – i.e. plot (5, 16), (10, 54), (20, 84), (30, 93) and (50, 100).



Increasing curve through 5 points [1]
5 points at upper ends of intervals on correct vertical lines [1] Points at 5 correct heights [1]

10c

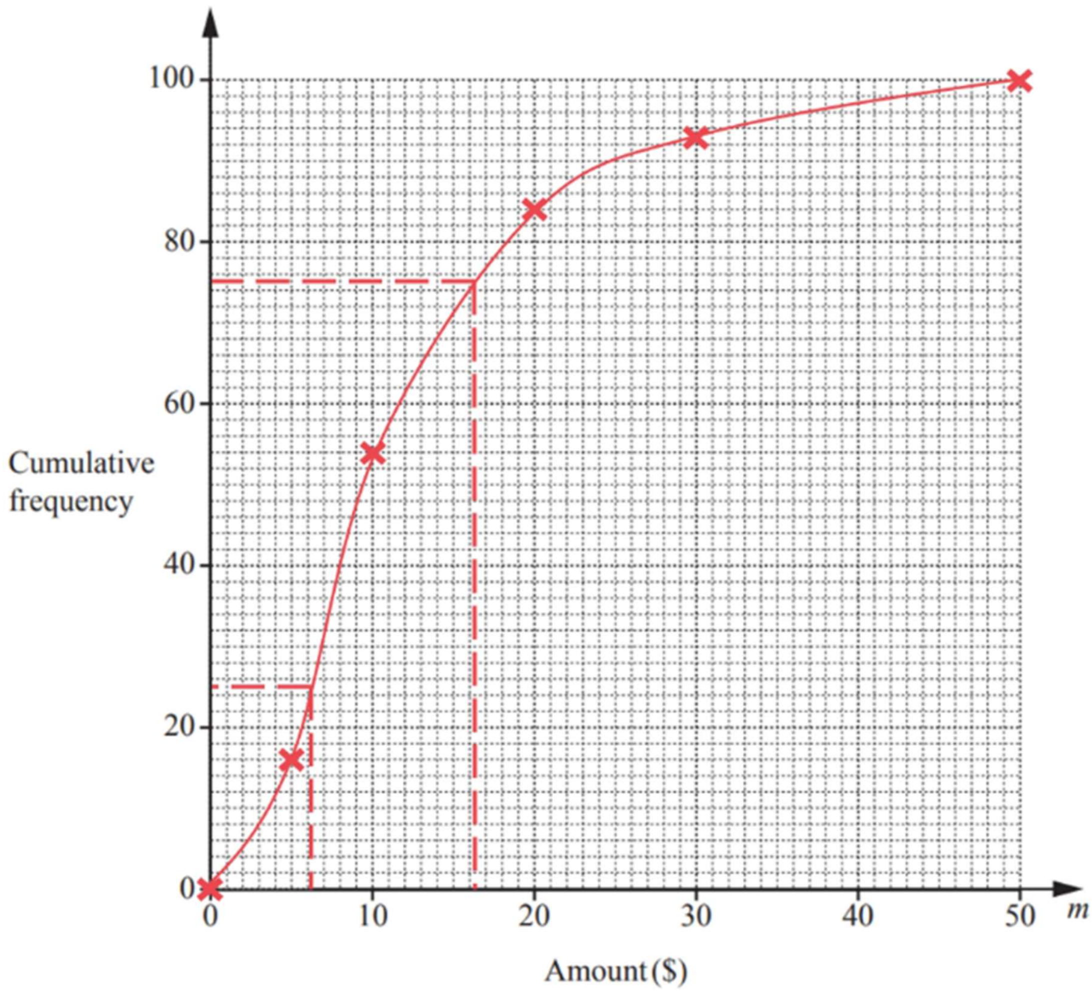
i) The median can be estimated by the $100 \div 2 = 50^{th}$ data value. Draw a horizontal line at 50 on the cumulative frequency axis across to the curve, then a horizontal line down to the amount axis.
Read off the value for an estimate of the median.



$ 9.50 [1]
9 to 9.8 allowed

ii) The interquartile range (IQR) is the difference between the upper quartile (UQ) (three-quarters of the way through the data) and the lower quartile (LQ) (one-quarter of the way through the data)
As there are 100 pieces of data, these can be estimated by the 75^{th} and 25^{th} pieces of data respectively.
Draw horizontal lines from 75 and 25 on the cumulative frequency axis across to the curve; then draw a vertical line down to the amount axis.

+918591171615 https://pointsedulab.com/ info@pointsedulab.com



Read off the amount axis to estimate the UQ and LQ.

UQ = 16.5
LQ = 16.1

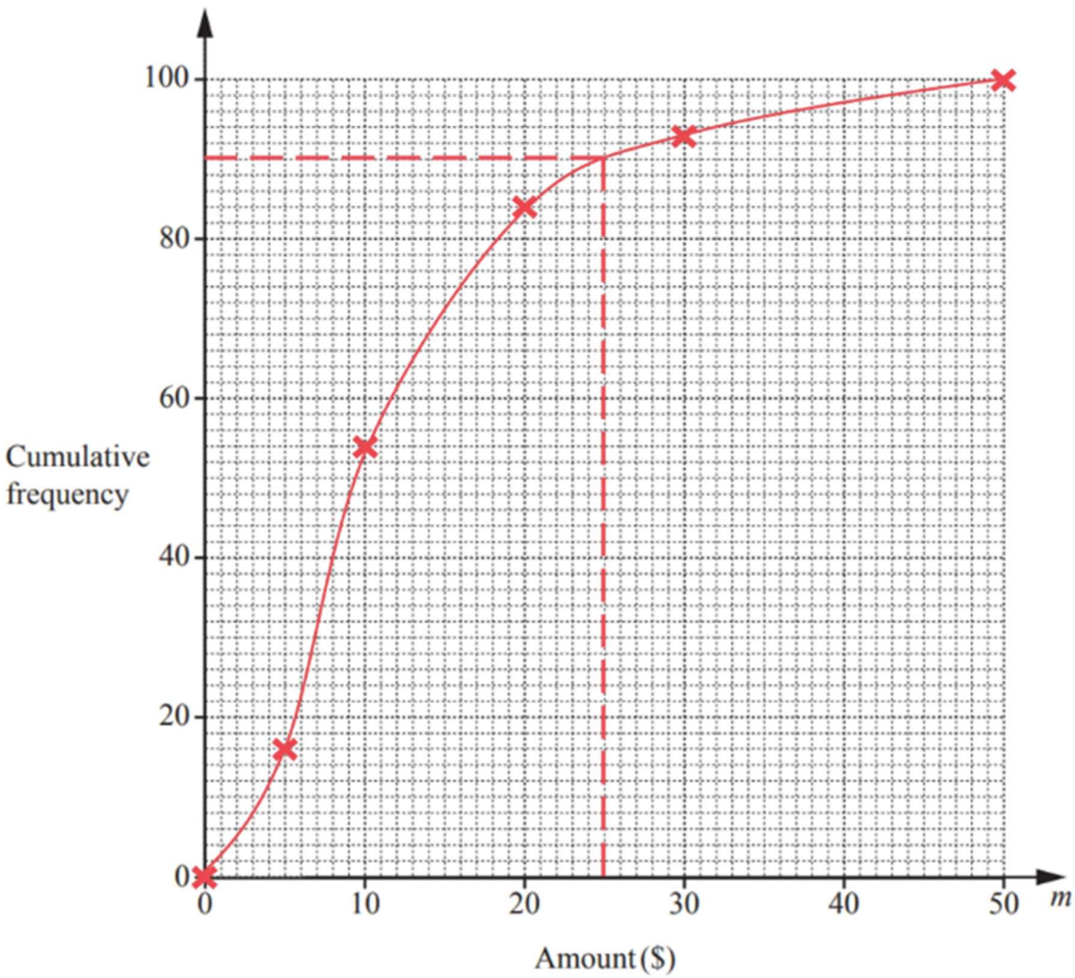
UQ between 15.5 and 17.5, or LQ between 6 and 7 [1]

Find the interquartile range, IQR = UQ - LQ.

$$16.5 - 6.1 = 10.4$$

10.4 [1]
8.5 to 11.5 allowed

iii) Use the cumulative frequency graph to estimate the number of students who spent **less** than $25.
Draw a vertical line up from $25 on the amount axis to the curve and then draw a horizontal line across to the cumulative frequency axis.



Read off the cumulative frequency axis to estimate the number of students who spent **less** than $25.

90 students spent $25 or **less**

As there are 100 students in total, we can find the number who spent **more** than $25 by subtraction.

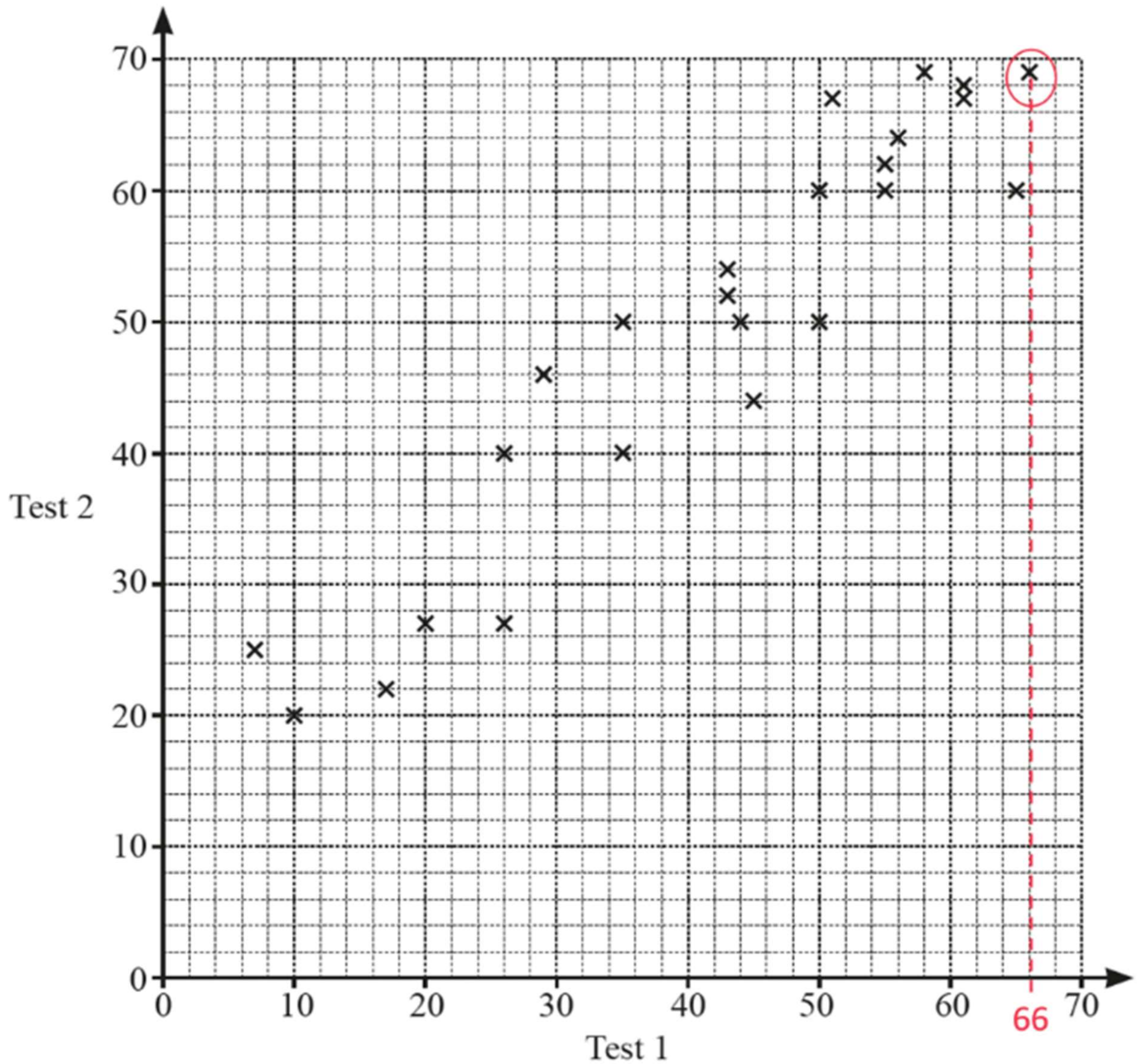
$$100 - 90 = 10$$

10 students [2]
10, 11, or 12 allowed due to graph accuracy

+918591171615 https://pointsedulab.com/ info@pointsedulab.com

11a

On the scatter diagram, identify the furthest point to the right, as this will be the highest on the Test 1 axis.



66 marks [1]

11b

As Test 1 scores increase, Test 2 scores tend to increase. So we have positive correlation.

Positive correlation

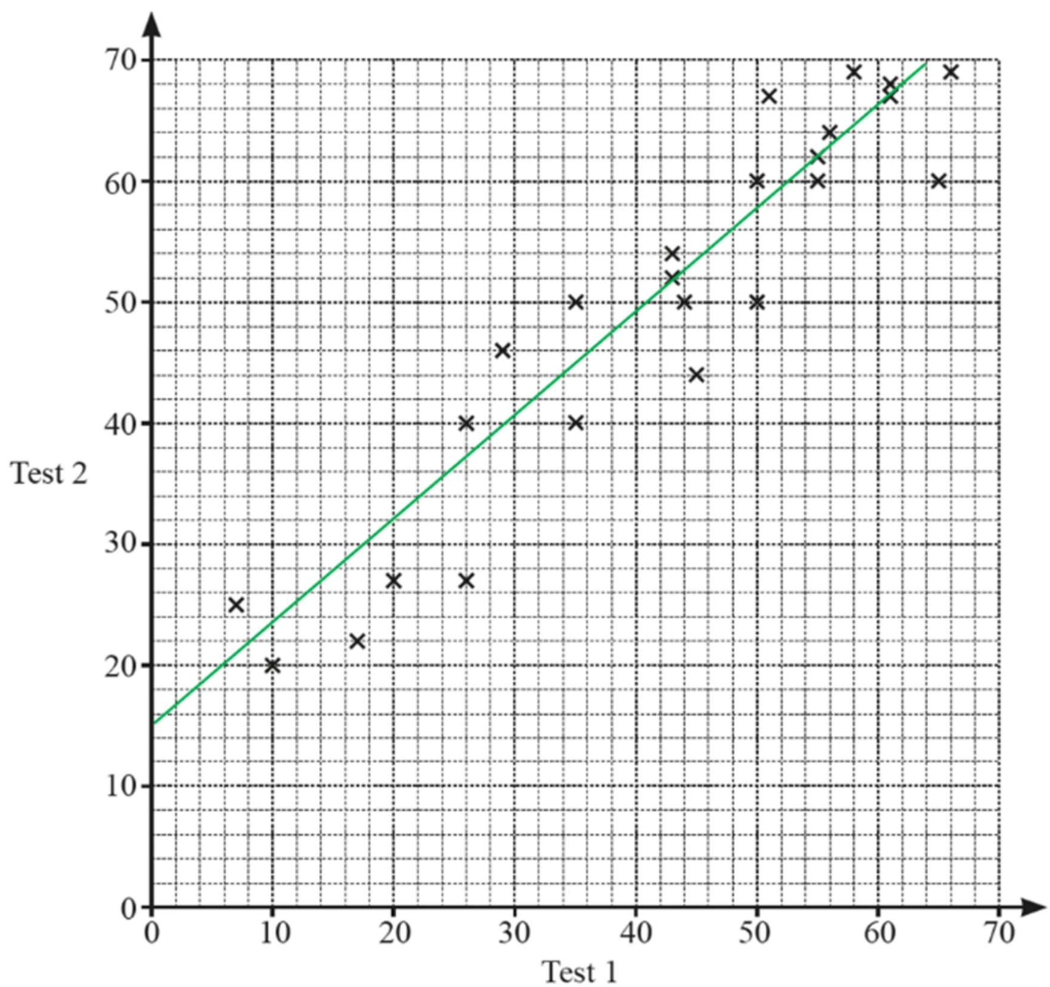
the word "positive" [1]

11c

The line of best fit (LOBF) is drawn by eye so there is some margin of error.
The LOBF does not have to, but may, pass through any of the plotted points.
In this case, the LOBF does not necessarily have to pass through the origin.

The LOBF must be drawn with a ruler/ straight edge and its gradient must match the correlation of the points (positive/ negative). It should have roughly the same number of points above as below it and should extend all the way to the edges of the grid.

The LOBF drawn below is an example of an appropriate answer.

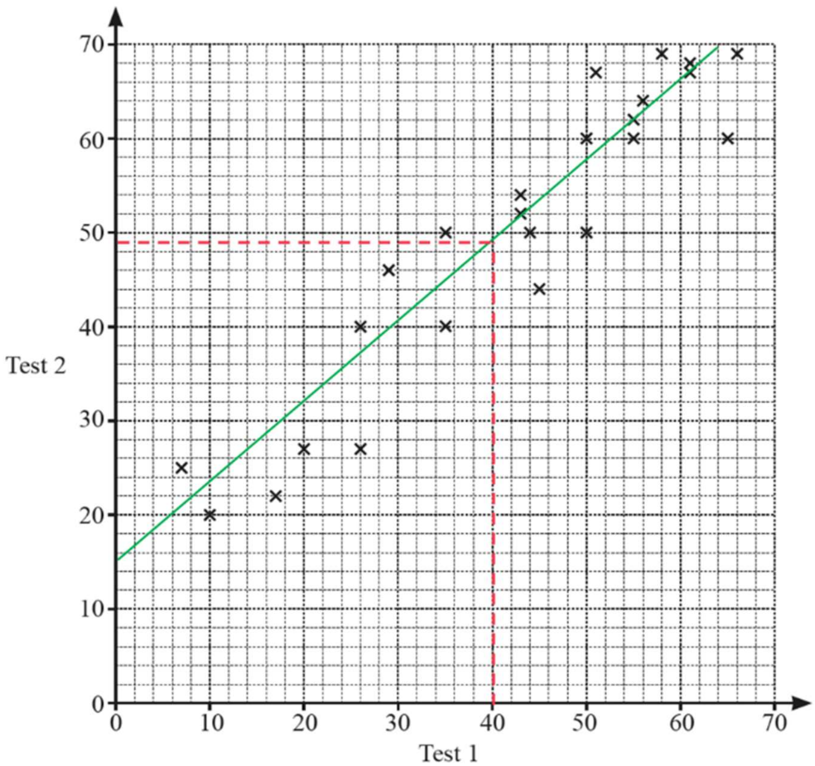


correct LOBF drawn with a straight edge [1]

11d

Your answer to this question will depend on your line of best fit (LOBF) drawn in the previous part of the question. But your method should always be the same.

Draw a vertical line up from 40 on the Test 1 axis to your LOBF. Then draw a horizontal line across to the Test 2 axis.



Read off the Test 2 axis to estimate the score.
According to *this* LOBF, the answer would be **49**.

46 to 50

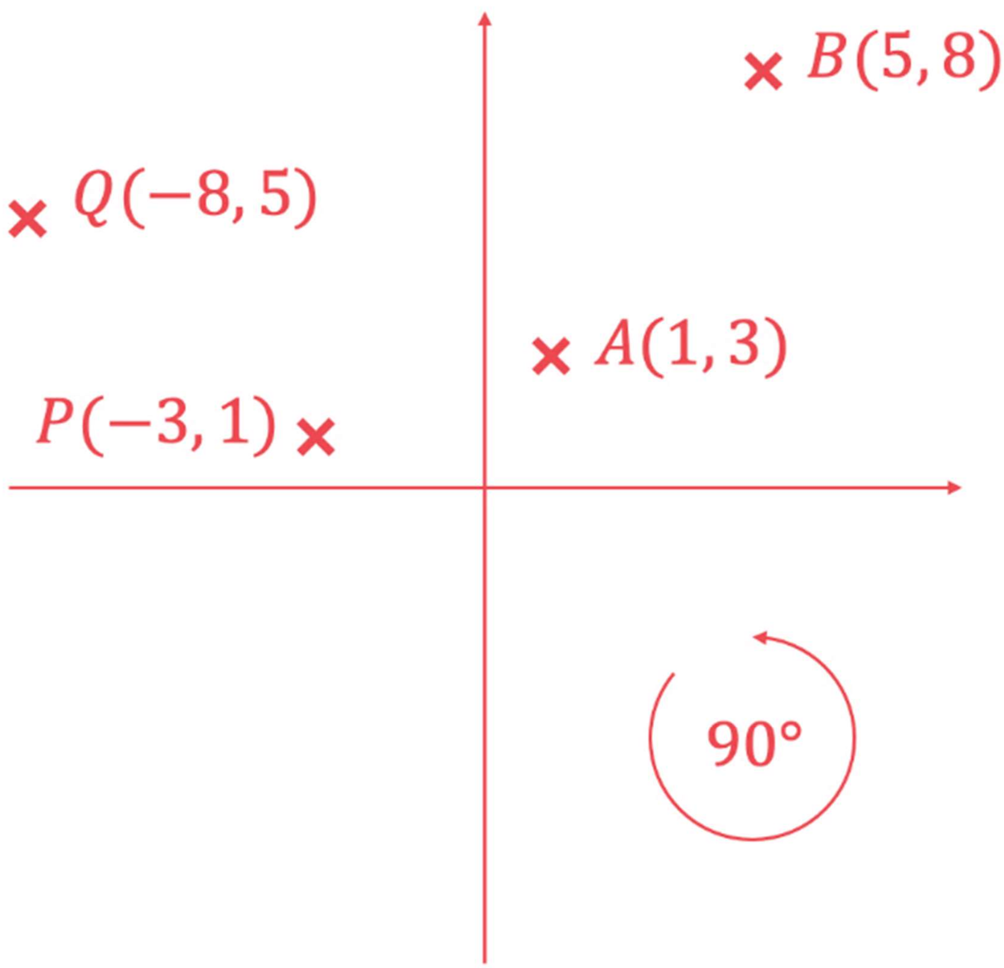
any answer between 46 and 50 that's consistent with the LOBF [1]

+918591171615 https://pointsedulab.com/ info@pointsedulab.com

12

i) Sketching a little diagram may help you to work out where the points will go.

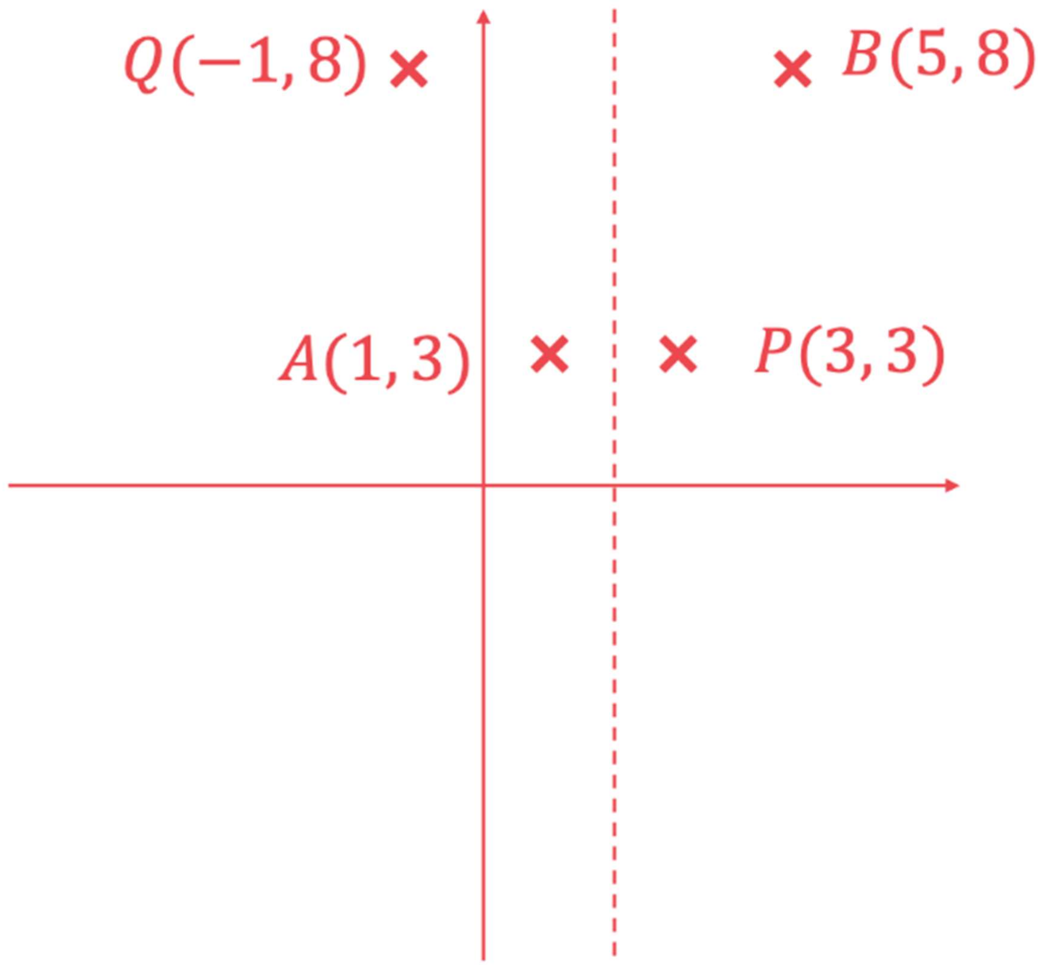
The x coordinate of the original object will become the y coordinate of the rotated image. The y coordinate of the original object will become the negative x coordinate of the rotated image.



$P(-3, 1)$
$Q(-8, 5)$

Two or three values correct [1]
The rest of the values correct [1]

ii) Sketching a little digram may help you to work out where the points will go.
Draw in the line $x = 2$ and work out how many steps away from the line the x coordinates of A and B are, then count them on the other side of the line to find the position of P and Q.
The y values will remain the same.



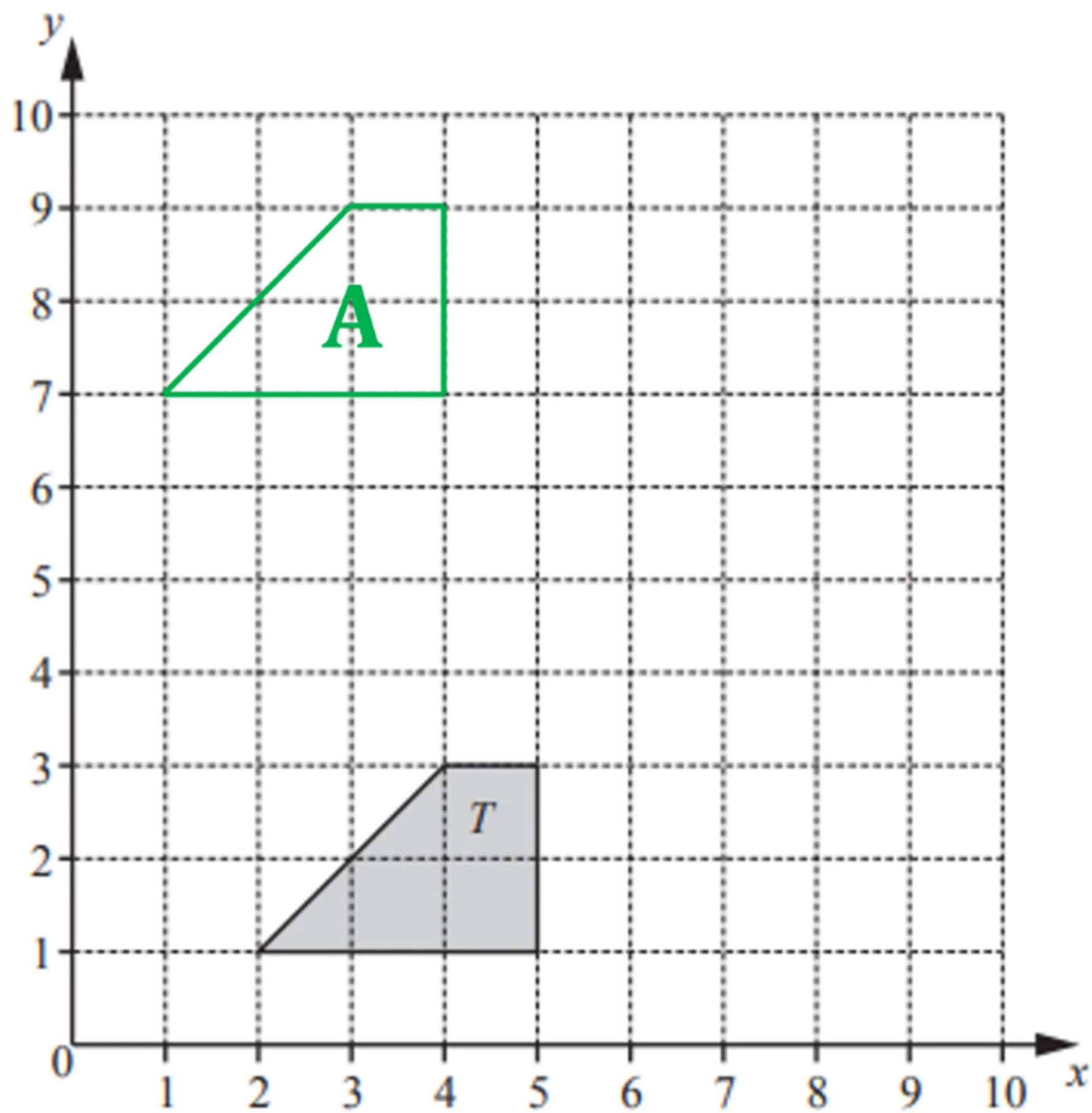
$P(3, 3)$
$Q(-1, 8)$

Two or three values correct [1]
The rest of the values correct [1]

+918591171615 https://pointsedulab.com/ info@pointsedulab.com

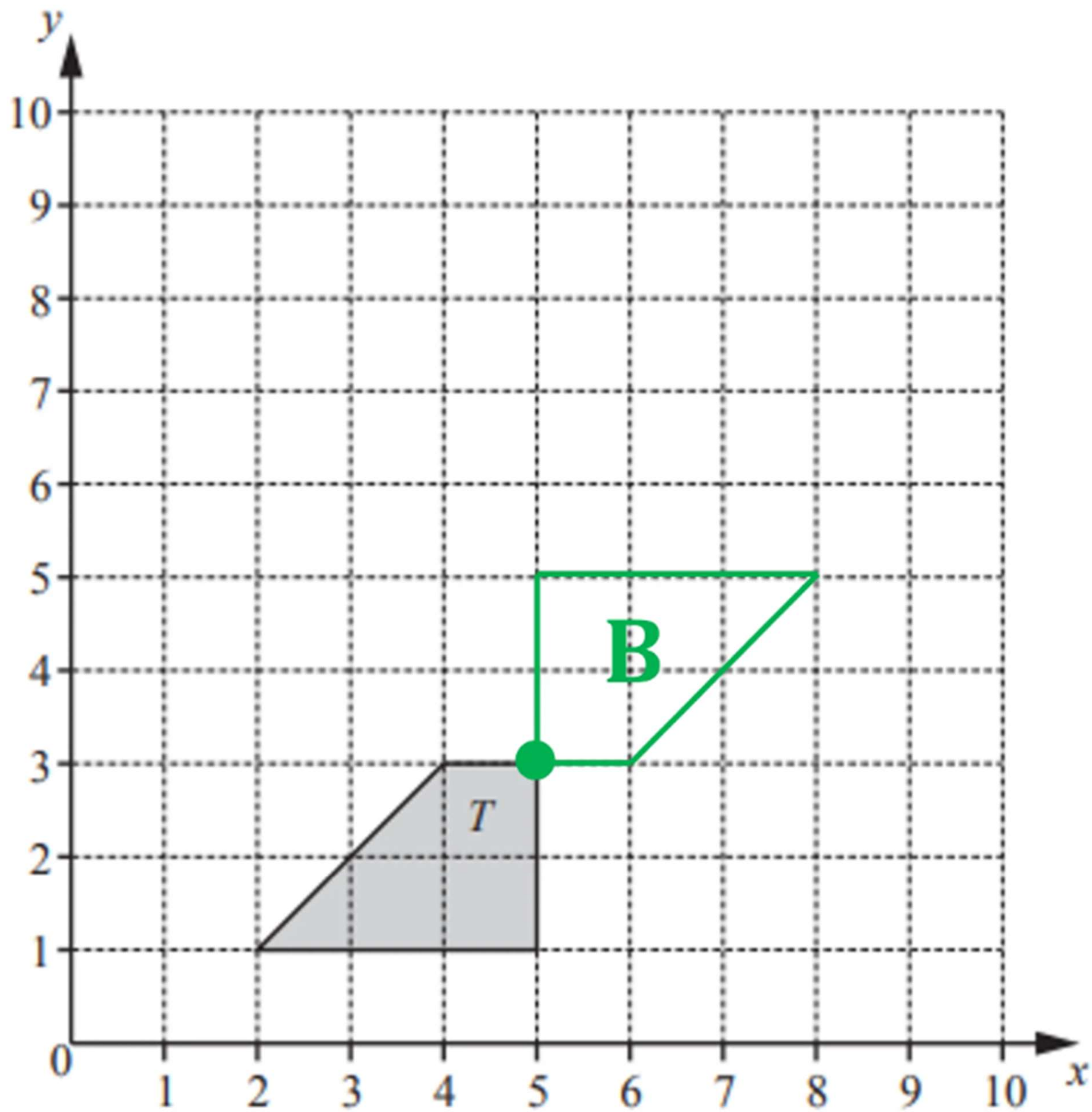
13

i) 'Slide' the shape 1 unit left, and 6 units up



Translating 1 left [1]
Translating 6 up [1]

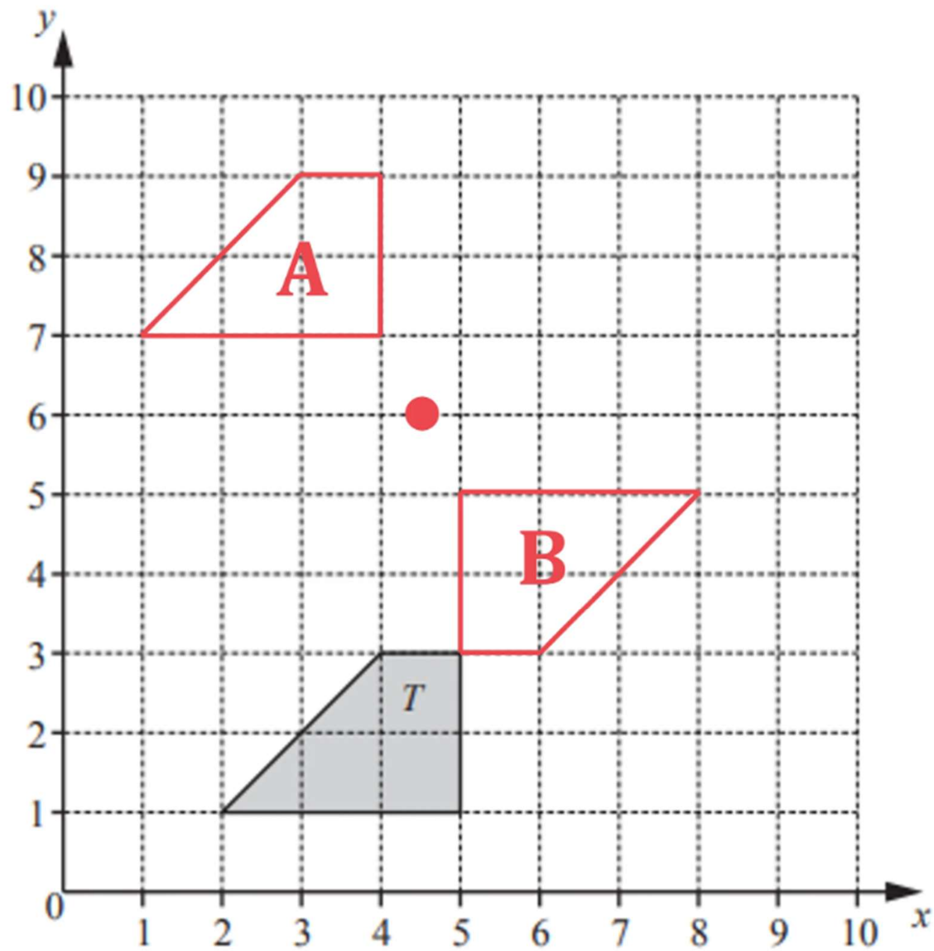
ii) Mark (5,3) on the grid, and make use of tracing paper Trace the image, keep your pencil fixed on (5,3) to 'pin' the tracing paper Rotate the paper 180°, and redraw the shape on the grid



Rotation by 180°, using wrong centre [1]
Fully correct [1]

+918591171615 https://pointsedulab.com/ info@pointsedulab.com

iii) Find the point halfway between two corresponding (matching) vertices; (4.5,6)



There are two possibilities for how *A* maps to *B*; either a rotation of 180° or an enlargement of scale factor -1

Either:

Rotation [1]

by 180° [1]

about the point (4.5,6) [1]

Or:

Enlargement [1]

by scale factor -1 [1]

with centre of enlargement (4.5,6) [1]

14

$\left|\overrightarrow{MT}\right|$ means magnitude of a vector. The formula for the magnitude of a vector is

$$\left|\begin{pmatrix} a \\ b \end{pmatrix}\right| = \sqrt{a^2 + b^2}$$

Substitute the components of the vector into the formula for the magnitude.

$$\sqrt{(2k)^2 + (-k)^2} = \sqrt{180}$$

[1]

Get rid of the square root by squaring both sides.

$$(2k)^2 + (-k)^2 = 180$$
$$4k^2 + k^2 = 180$$
$$5k^2 = 180$$

[1]

Solve.

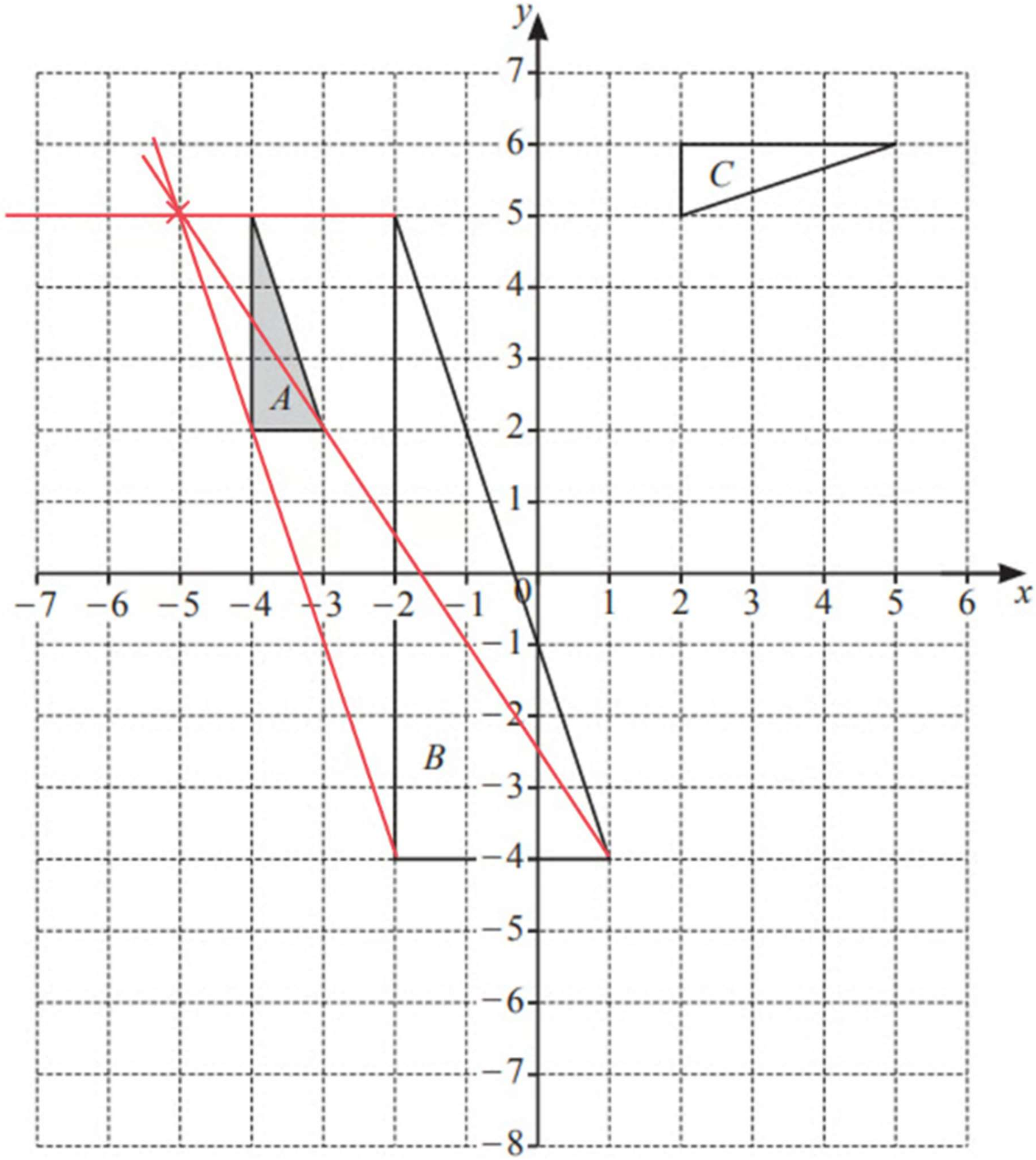
$$k^2 = 36$$
$$k = \pm 6$$

Positive value of $k = 6$ [1]

15

i) *B* is a different size to *A* so it must be an enlargement.
The horizontal base of triangle *B* is 3 units and the horizontal base of triangle *A* is 1 unit so *A* has been enlarged by a scale factor of 3.

To find the centre of enlargement, use a ruler to draw lines from each vertex of *B* through the corresponding vertex of *A*.
The centre of enlargement will be the coordinates of the point at which the lines meet.

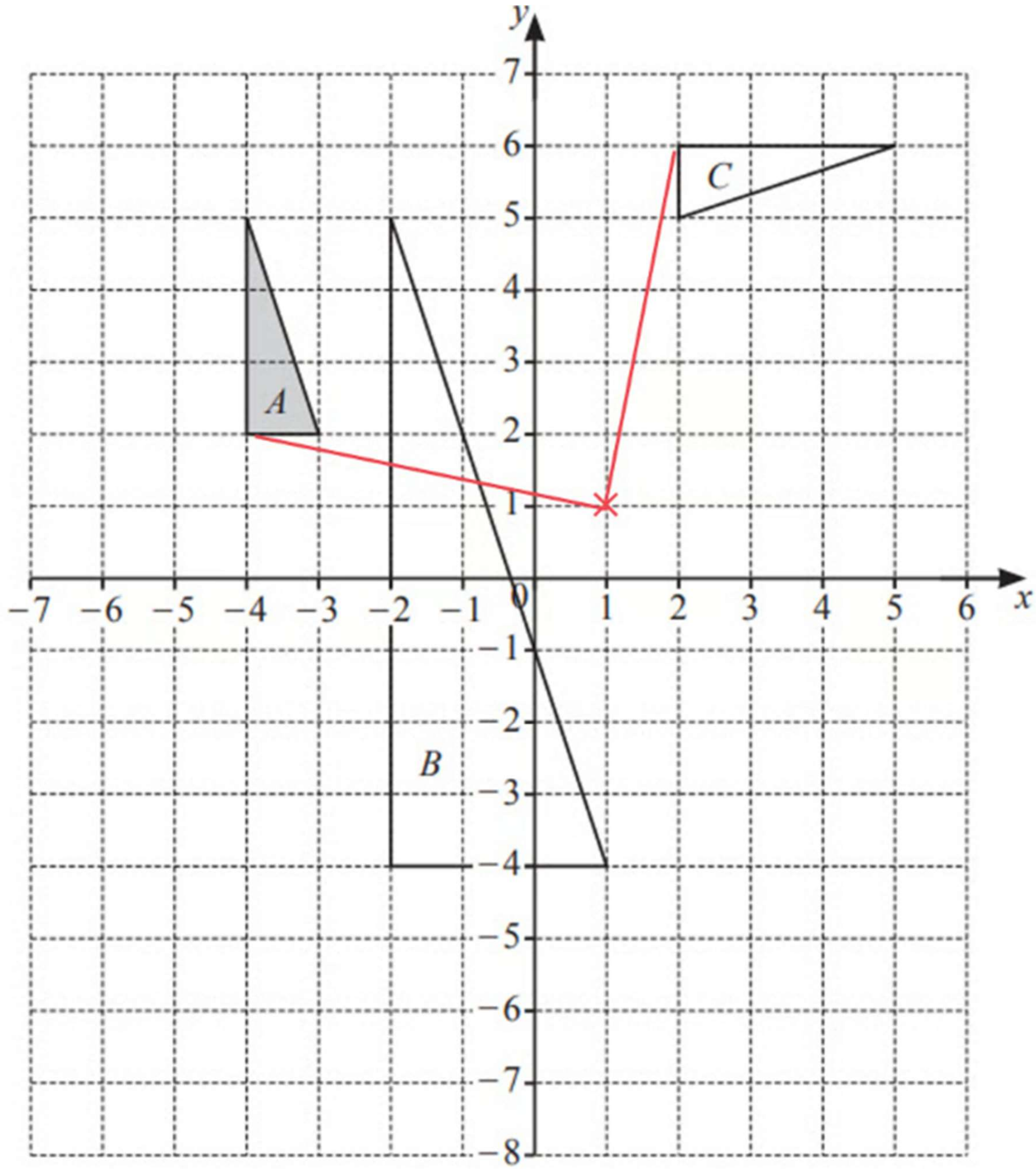


y
x
O
A
B
C
−7
−6
−5
−4
−3
−2
−1
1
2
3
4
5
6
7
−8

An enlargement [1]
Scale factor 3 [1]
Centre of enlargement (−5, 5) [1]

a) Triangle *A* has been turned so it must be a rotation.
Triangle *A* has made a quarter turn in the clockwise direction, so it has rotated 90° clockwise.

To find the centre of rotation, use tracing paper to try different centres of rotation.
Trace over triangle *A* and then put the point of your pencil at different points and rotate the tracing paper 90° clockwise until your drawing fits perfectly on C.



Rotation [1]

90° clockwise [1]

Centre of rotation (1, 1) [1]

A rotation of 270° anticlockwise about the point (1,1) is also accepted.

16

Substitute **a** , **b** and **c** into the formula

$$m\begin{pmatrix}-3\\2\end{pmatrix}+n\begin{pmatrix}5\\4\end{pmatrix}=\begin{pmatrix}14\\9\end{pmatrix}$$

Multiply any letters on the outside of a vector by the two numbers inside

$$\begin{pmatrix}-3m\\2m\end{pmatrix}+\begin{pmatrix}5n\\4n\end{pmatrix}=\begin{pmatrix}14\\9\end{pmatrix}$$

Add the two vectors on the left (by adding the tops and bottoms separately)

$$\begin{pmatrix}-3m+5n\\2m+4n\end{pmatrix}=\begin{pmatrix}14\\9\end{pmatrix}$$

Form simultaneous equations (by reading off the top row and the bottom row)

$$-3m+5n=14$$
$$2m+4n=9$$

[1]

These simultaneous equations can be solved by eliminating m (or n) Make the m terms equal in size (by multiplying the first equation by 2 and the second equation by 3)

$$-6m+10n=28$$
$$6m+12n=27$$

[1]

Add these two equations by adding "like" (similar) terms The m terms should disappear

$$22n=55$$

Solve this equation for n (by dividing both sides by 22)

$$n = \frac{55}{22} = \frac{5}{2}$$

[1]

Substitute $n = \frac{5}{2}$ back into the first equation (or the second)

$$-3m + 5 \times \frac{5}{2} = 14$$
$$-3m + \frac{25}{2} = 14$$

[1]

Solve this equation for m (for example, by subtracting $\frac{25}{2}$ from both sides then dividing by -3)

$$-3m = 14 - \frac{25}{2}$$
$$-3m = \frac{3}{2}$$
$$m = \frac{\frac{3}{2}}{-3} = -\frac{1}{2}$$

Check $m = -\frac{1}{2}$ and $n = \frac{5}{2}$ satisfy the other equation ($2m + 4n = 9$) by substituting them into the left-hand side

$$2 \times \left(-\frac{1}{2}\right) + 4 \times \frac{5}{2}$$
$$= -1 + 10$$
$$= 9$$

Give both answers together at the end

$m = -\frac{1}{2}$ and $n = \frac{5}{2}$ [1]