POINTS®

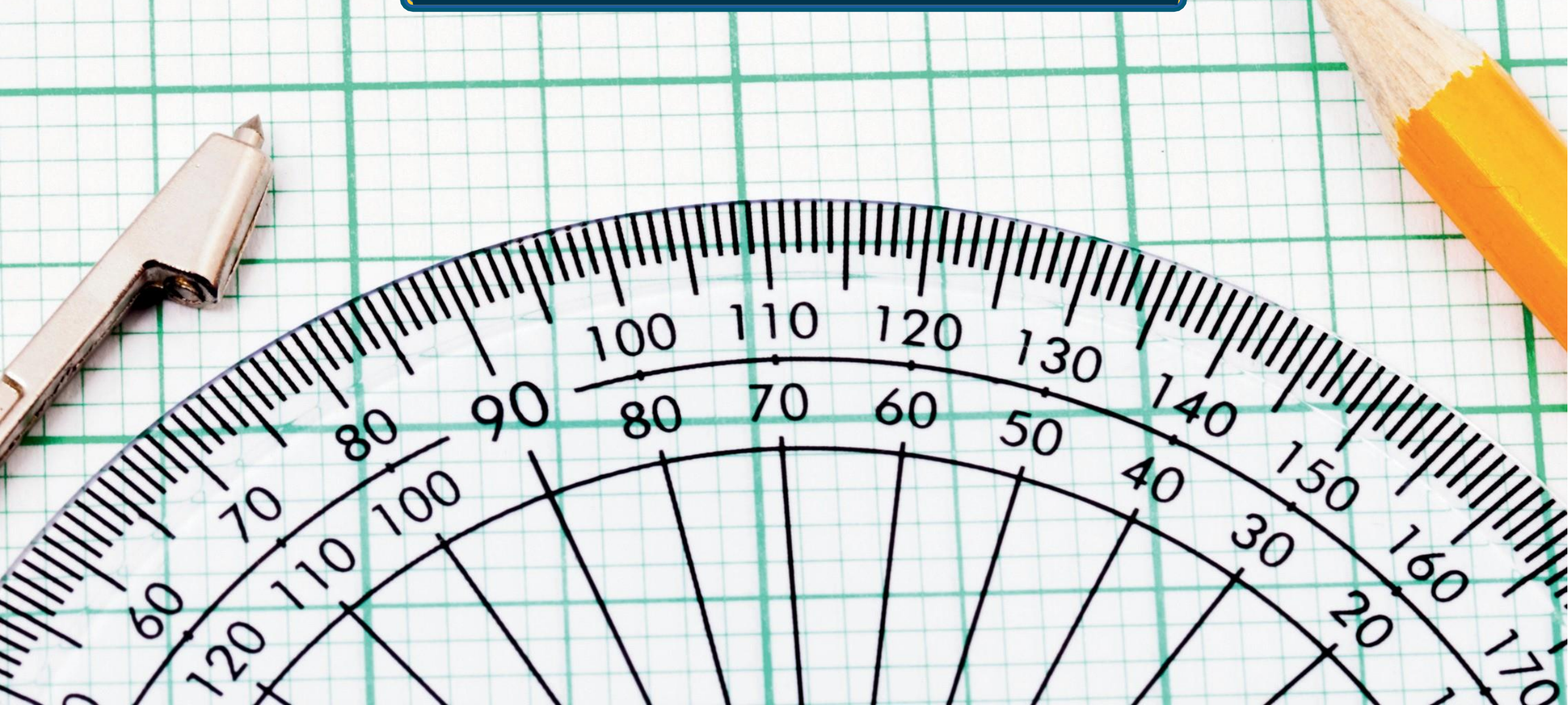
Your International Curriculum Expert

ELEVATE

MATH TOPICAL WORKSHEETS

IGCSE-International Mathematics (0607)

Circle theorem,Mensuration &Trigonometry



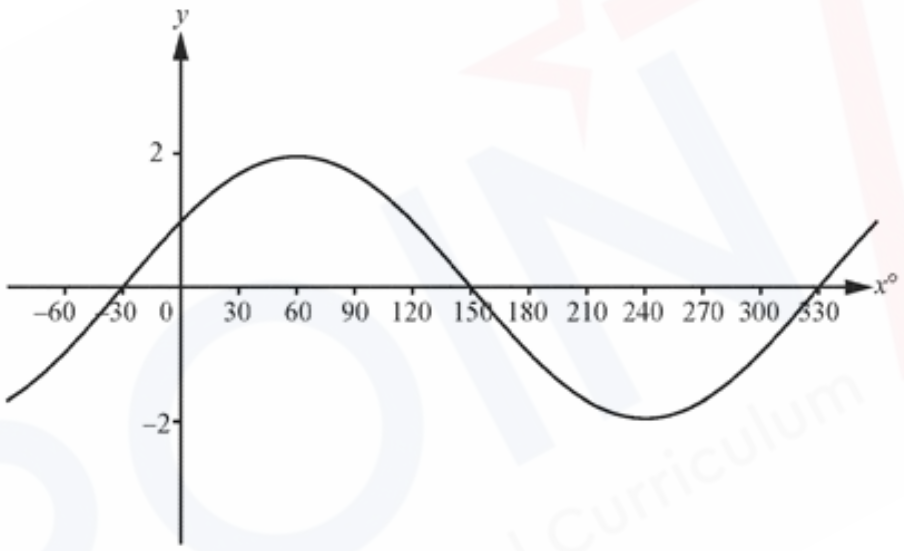
❖ Paper-2 (QP)

1 The graph of y = acos(bx)^0 has a maximum point at (360, 3) and a minimum point at (450,-3).

Find the value of a and the value of b,

$answer\ a = \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.$
$b = \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\ [2]$

2.



The graph of $y = asin\,(x + b)^0$ is shown in the diagram.

Find the value of a and the value of b.

$answer\ a = \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.$
$b = \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\ [2]$

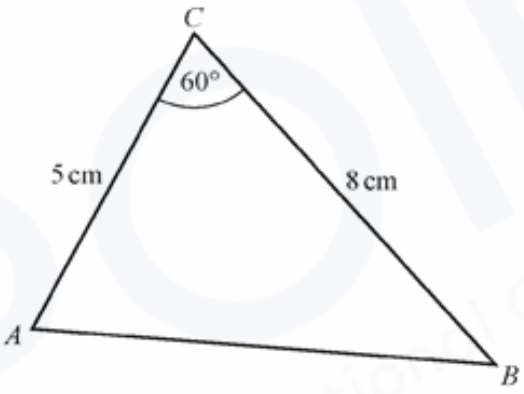
3. $f(x) = 6cos(6x)$

Find the amplitude and the period of f(x).

Amplitude = ...

Period = ...[2]

4.



NOT TO SCALE

Find AB.

AB = ... cm [3]

5. θ is an acute angle and $tan\theta = \sqrt{3}$

Write down the value of θ

$\theta = \cdots$ ……[1]

6. (a) On the grid, sketch the graph of $y = sinx^0, for\ 0 \leq x \leq 360.$

[2]

(b) The point (a, 0.5) is on the graph of $y = sinx^0$

Find the two possible values of a.

a = ... or a=..... [2]

7. $\sin\theta = -\frac{1}{\sqrt{2}}$ *and* $0 \leq x \leq 360$

Find the two value of θ

$\theta = \cdots \ldots \ldots \ldots \ldots \ldots \ldots$, *or* $\theta = \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$ [2]

8. $f(x) = 3\,sin\,2x^0$

(a) Write down the amplitude of the graph of f(x).

.....[1]

(b) The graph of y = f(x) goes through the points (75, 1.5) and (a. 1.5).

Find a possible value of a greater than 75.

.....[1]

9. The table shows some trigonometric ratios, each correct to 3 decimal places.

	Sine	**Cosine**	**Tangent**
40°	0.643	0.766	0.839
70°	0.940	0.342	2.747

Use this information to find

(a) sin110°

....[1]

(b) tan320°

....[1]

10. α is acute and $tan\ \alpha = x$.

Find, in terms of x,

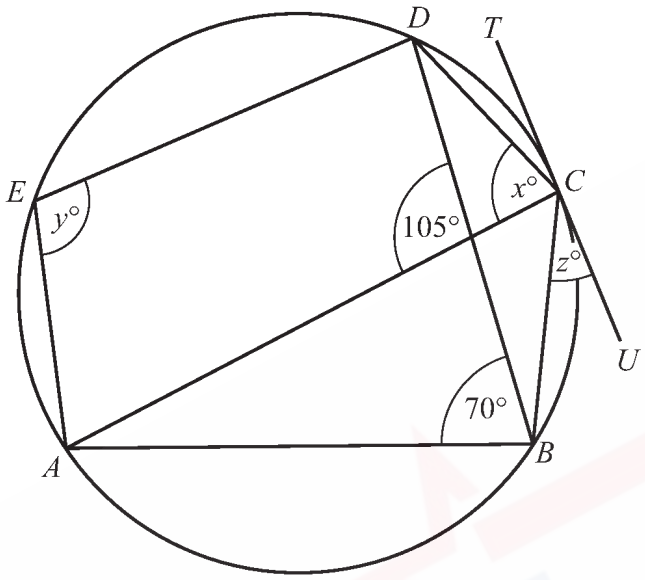
$(\boldsymbol{a})$ $tan(180 - \alpha)$,

$tan(180 - \alpha) = \cdots$ ... ... [1]

$(\boldsymbol{b})$ $tan(90 - \alpha)$,

$tan(90 - \alpha) = \cdots$ ... ... [1]

11



NOT TO SCALE

A, B, C, D and E lie on the circle.
TU is a tangent to the circle at C.

Find the values of x, y and z.

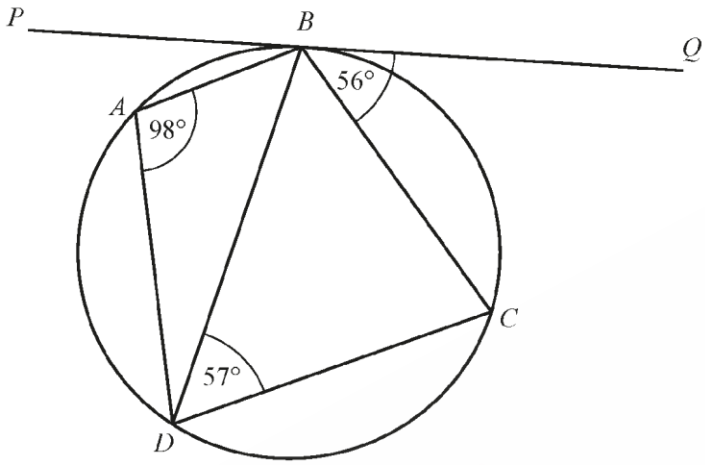
$x =$ .....

$y =$ .....

$z =$ .....

[4]

12



NOT TO SCALE

A, B, C and D are points on the circle.
PBQ is a straight line.

(a) Find angle DCB, giving a reason for your answer.

Angle DCB = ... because .....

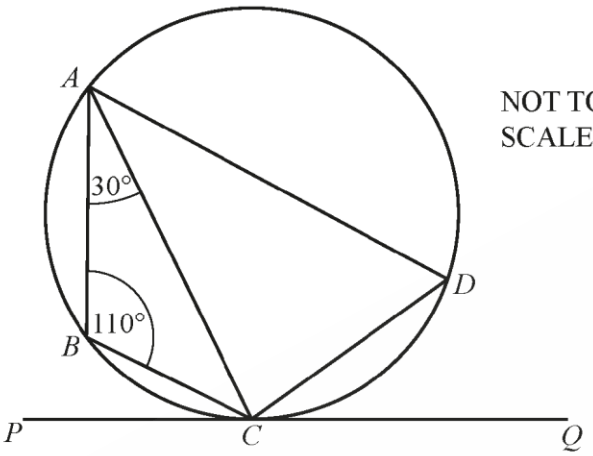
..... [2]

(b) Is PBQ a tangent to the circle?
Give a reason for your answer.

..... because .....

..... [1]

13



NOT TO SCALE

The points A, B, C and D lie on a circle.
PCQ is a tangent to the circle at C.
Angle $ABC = 110°$ and angle $BAC = 30°$.

Find

(a) angle ADC,

Angle ADC = ... [1]

(b) angle ACP,

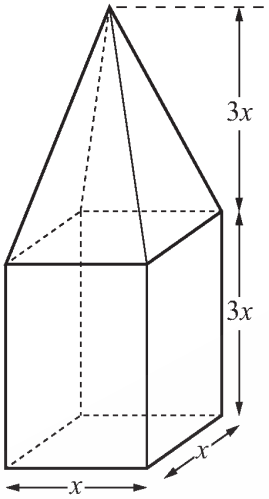
Angle ACP = ... [1]

(c) angle PCB.

Angle PCB = ... [1]

14

In this question all lengths are in centimetres.



NOT TO SCALE

A solid is made by joining a cuboid to a pyramid.
The base of the cuboid is a square of side x.
The height of the cuboid is $3x$.

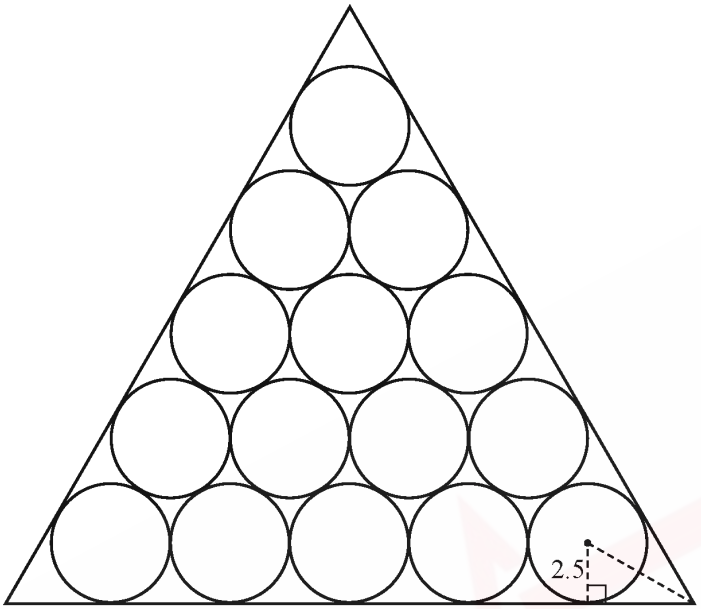
The base of the pyramid is a square of side x.
The height of the pyramid is $3x$.

The total volume of the solid is $32\,000\ \text{cm}^3$.

Show that $x = 20$.

[3]

15



NOT TO SCALE

The diagram shows 15 circles in an equilateral triangle.
The circles touch each other and the triangle.
The radius of each circle is 2.5 cm.

The length of each side of the triangle is $\left(a+b\sqrt{3}\right)$ cm where a and b are integers.

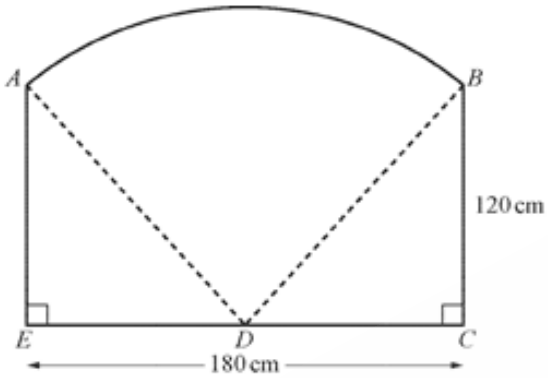
Find the value of a and the value of b.

$a =$ .....

$b =$ ..... [5]

❖ Paper 4

1.



The diagram shows a fence panel ABCDE.

The vertical edges AE and BC are of length 120 cm and the horizontal base EC is of length 180 cm.

D is the midpoint of EC.

(a) Calculate AD.

Answer (a) .....cm [2]

(b) Show that angle ADB = 73.74° correct to 2 decimal places.

[3]

(c) AB is an arc of a circle centre D.

Find the area of the fence panel.

Answer(c) ..... cm^2 [3]

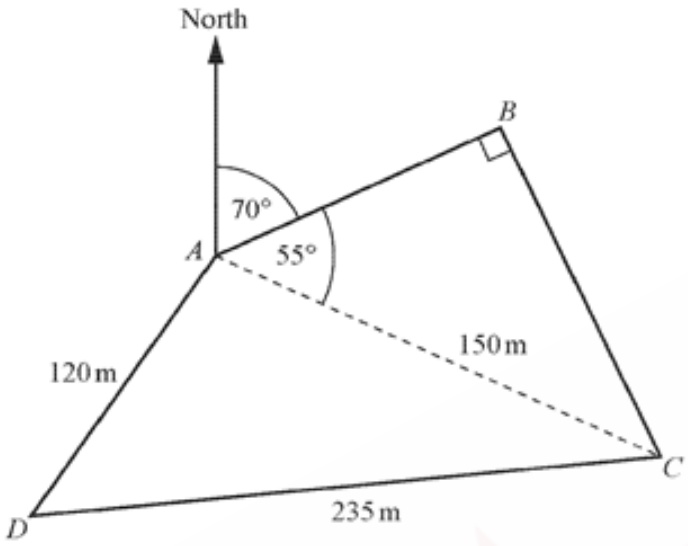
(d) Stefan's fence has 8 panels, each identical to ABCDE

He wishes to paint both sides of all the panels each litre of paint covers an area of 6 square metres

Calculate the number of litres Stefan needs to paint both sides of the whole fence.

Answers (d).....litres **[3]**

2.



The diagram shows a field ABCD with a path from A to C AC = 150m, AD = 120 m and CD = 235 m. Angle ABC = 90°, angle BAC = 55° and the bearing of B from A is 070°.

(a) Calculate the length of AB.

Answers (a)..... m [2]

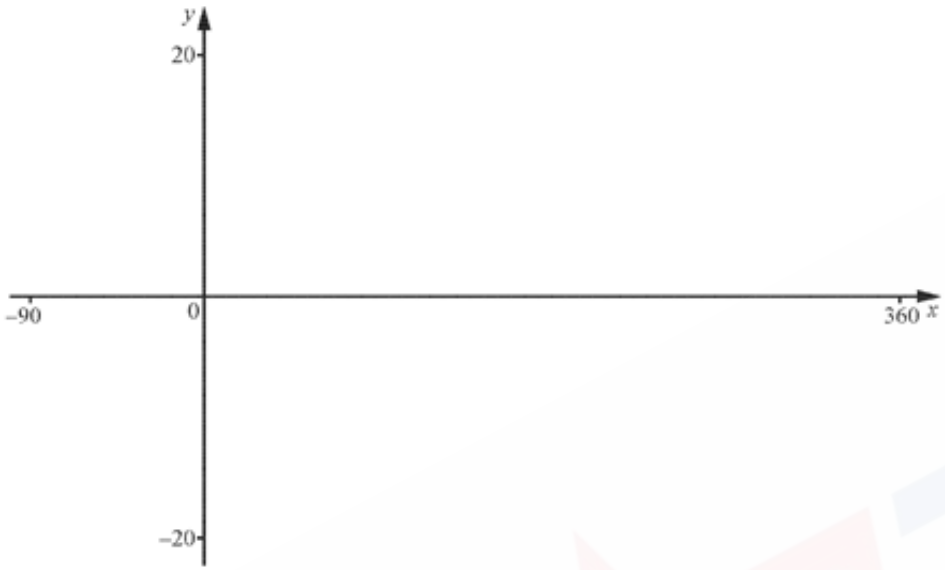
(b) Calculate the bearing of D from A.

Answer (b)..... [4]

(c) Calculate the area of the field ABCD.

Answer(c) ..... cm^2 [3]

3.



$f(x) = 2tan(x + 30)^0$

(a) On the diagram, sketch the graph of y=-f(x) for values of x between 90 and 360.

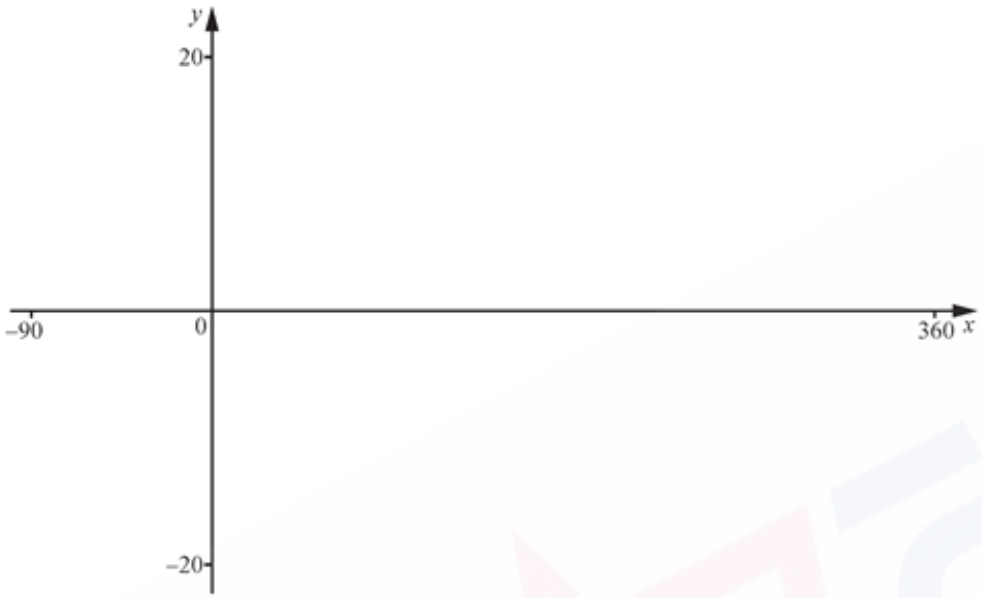
(b) Solve the equation f(x) = 5 for values of x between -90 and 360.

Answers (b) x=..... or x=..... [2]

(c) Write down the equations of the two asymptotes to this graph for values of x between-90 and 360.

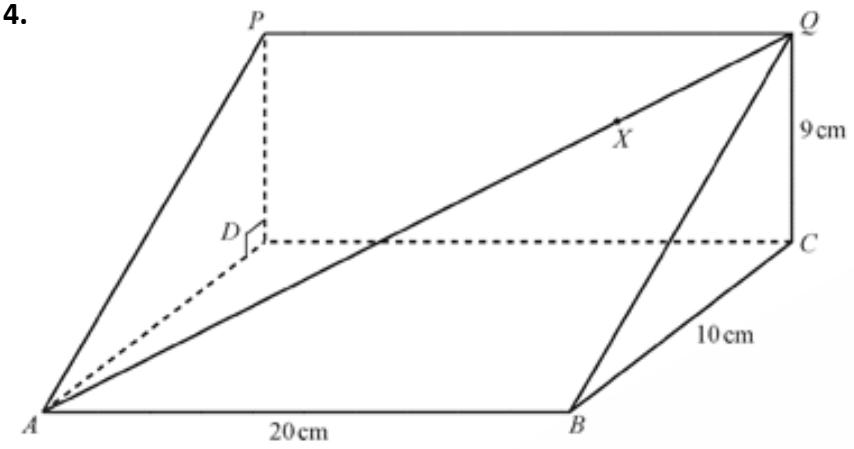
Answer(c).....[2]

(d) On the diagram below, sketch the graph of $y = |2tan(x + 30)^0|$ for values of x between -90 and 360.



[2]

4.



NOT TO SCALE

The diagram shows a triangular prism with a horizontal base ABCD.

X is a point on the line 40.

AB= 20cm, BC = 10cm, CQ =9cm and angle BCQ = 90°

(a) Calculate angle OBC.

Angle QBC

.....[2]

(b) Calculate angle BAQ and show that it rounds to 33.9°, correct to 1 decimal place.

[3]

(c) AX= 22 cm

Calculate the length of BX.

BX = ... cm [3]

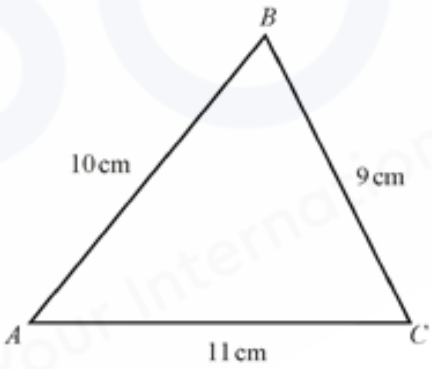
5a. $cosx = \frac{1}{3}\ for\ 0^0 < x < 90^0$

Find the exact value of sinx.

Give your answer as a surd.

$sinx = \cdots$ ..... [3]

(b)



NOT TO SCALE

(i) Show that $cos\ B = \frac{1}{3}$

[2]

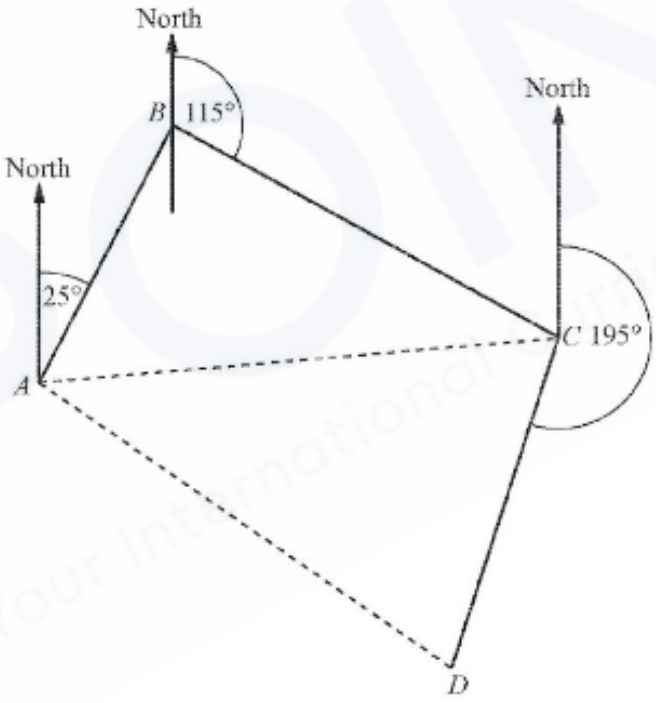
(ii) Using your answer to **part (a),** show that the exact value of the area of triangle ABC is$30\sqrt{2}\ cm^2$.

[3]

6. A ship sails on the following course.

- 60km on a bearing of 025° from A to B
- 80km on a bearing of 115o from B to C
- 75 km on a bearing of 1950 from C to D

The diagram shows the course.



NOT TO SCALE

(a) Show that angle ABC = 90°.

[1]

(b) Calculate angle BCA.

Angle BCA = ...[2]

(c) Calculate the distance AC.

AC = ... km [2]

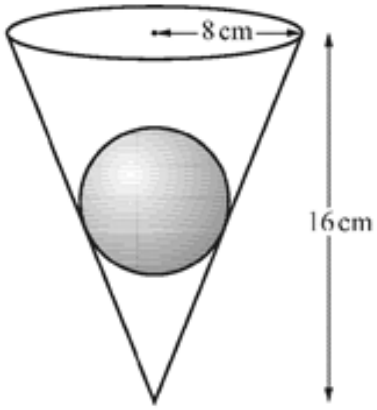
(d) Calculate the distance AD.

AD=..... km [4]

(e) Calculate the bearing of D from A.

.....[4]

7.



NOT TO SCALE

The diagram shows a solid sphere of radius 4 cm inside a hollow cone of radius 8 cm and height 16 cm. The sphere touches the interior of the cone.

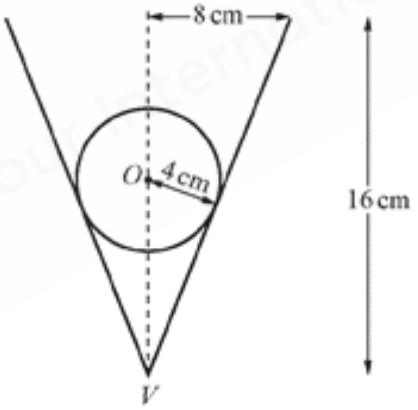
(a) Calculate the volume of the cone that is not occupied by the sphere.

..... m^3 [3]

(b) Calculate the curved surface area of the cone.

..... m^2 [3]

(c)



NOT TO SCALE

The centre, 0 of the sphere is directly above the vertex, V. of the cone,

Calculate the length OV

OV =..... cm [4]

8. A ship sails 65 km on a bearing of 310 from A to B.

It then changes course and sails 40 km on a bearing of 250° from B to C.

The ship then returns to A.

(a) On the diagram, sketch the path of the ship from A.

On your diagram show the bearings and distances.

[3]

(b) Find angle ABC,

.....[1]

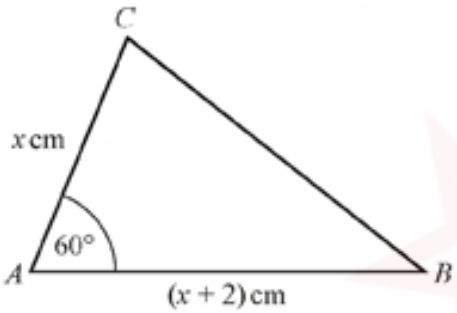
(c) Calculate AC and show that it rounds to 91.8 km, correct to the nearest tenth of a kilometre.

[3]

(d) Find the bearing of C from A.

..... [4]

9.



NOT TO SCALE

In the diagram AC = x cm, AB = (x + 2) cm and angle A = 60°.

(a) (i) Find an expression, in terms of x, for the area of triangle ABC.

Give your answer in surd form.

..... cm^2 [2]

(ii) The area of triangle $ABC = 18\sqrt{3}\ cm^2$.

Show that $x^2 + 2x - 72 = 0$.

[2]

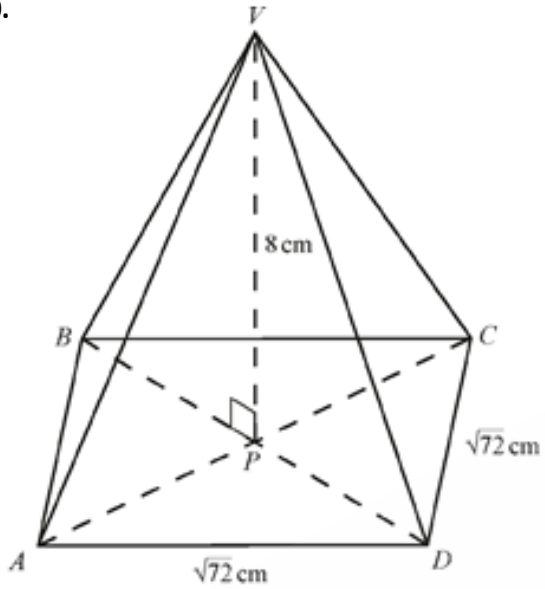
(b) (i) Solve the equation $x^2 + 2x - 72 = 0$.

x=…… or x=…… [2]

(ii) Find the shortest distance between the line AB and the point C.

.. … . . . . cm [2]

10.



NOT TO SCALE

The diagram shows a pyramid with a square base ABCD of side $\sqrt{72}$.

The diagonals of the base, AC and BD, meet at P.

The vertex, V. is vertically above P and VP = 8cm

(a) Find the volume of the pyramid.

Give the units of your answer

.....[3]

(b) Find the length AC.

AC=.....cm [2]

(c) Find the length DV.

DV = ... cm [3]

(d) Find angle VDP.

Angle VDP = ...[2]

(e) X is the midpoint of the side CD.

(i) Find the length VX.

VX = ...cm (3)

(ii) Find angle VXP.

Angle VXP =..... [2]

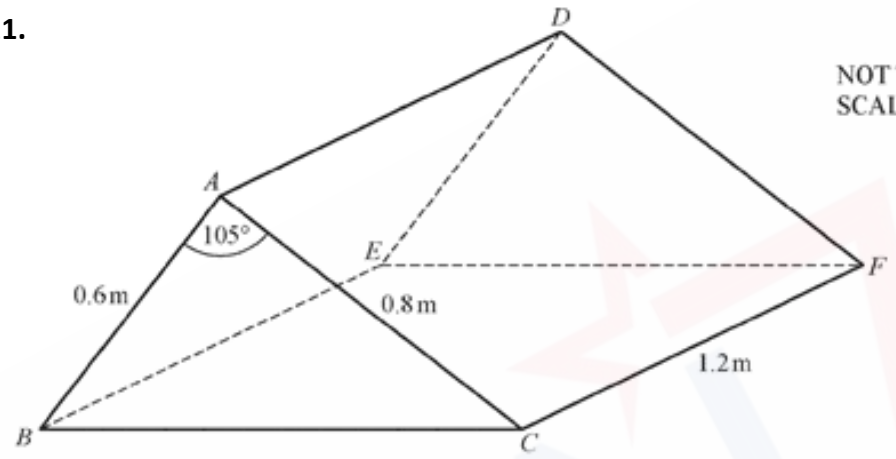
(e) The pyramid is cut parallel to ABCD to form a smaller pyramid VEFGH.

The volume of VEFGH is $24cm^3$.

Find the vertical height of this pyramid.

…. . . . . …. cm [3]

11.



ABCDEF is a solid triangular prism.

(a) Calculate the volume of the prism.

…….m^3[3]

(b) Calculate the total surface area of the prism.

…….m^2[5]

(c) ABCDEF is made of metal and has a mass of 2170kg

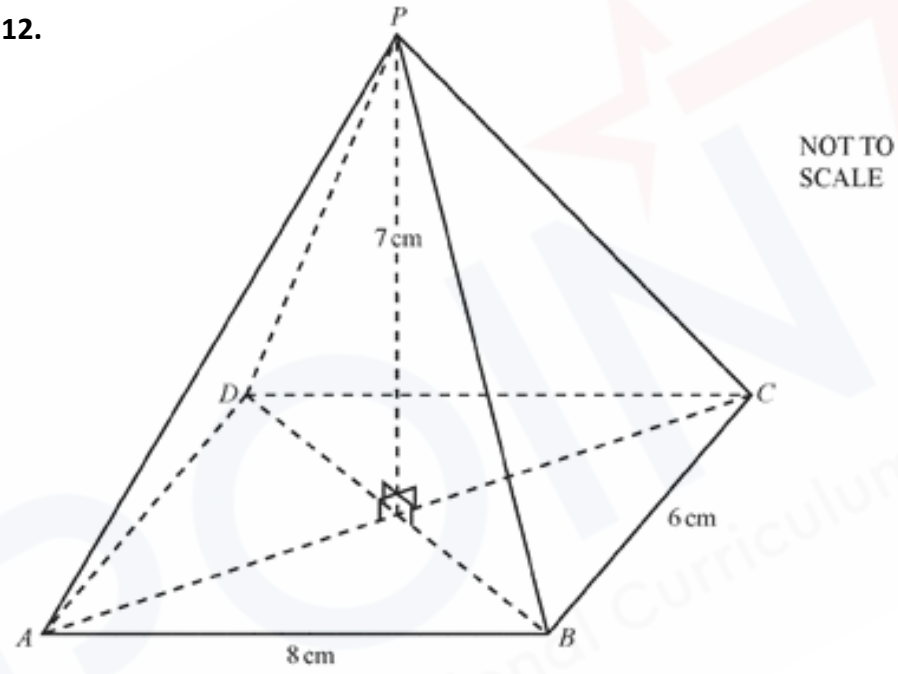
It is melted down and made into prisms similar to ABCDEF.

Each of these prisms has a mass of 2.17kg.

Calculate the total surface area of each of these smaller prisms.

.....m^2[3]

12.



The diagram shows a pyramid of height 7cm on a rectangular base 8 cm by 6cm .

The point P is directly above the centre of the base.

(a) Calculate the angle between the triangle PBC and the base ABCD.

.....[2]

(b) Calculate the angle between PB and the base ABCD.

.....[3]

(c) Calculate PC.

PC =..... cm [2]

(d) Calculate angle PCB.

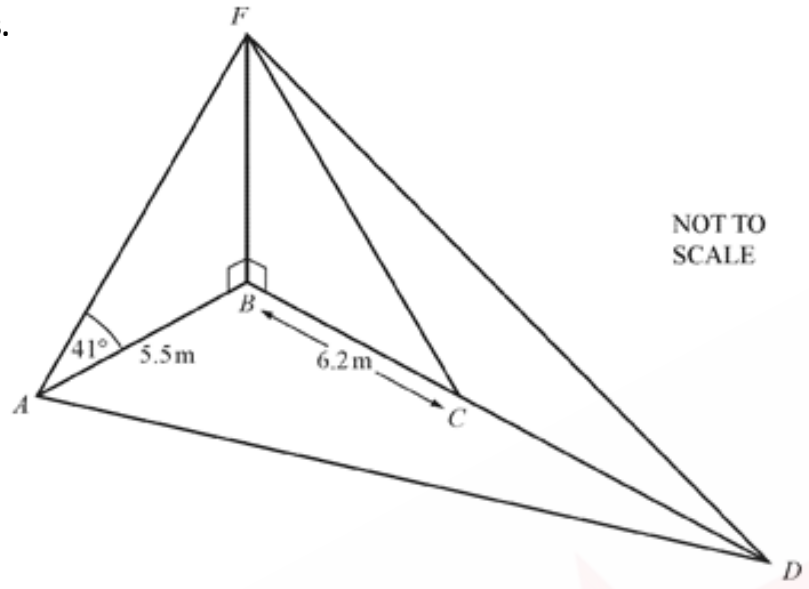
Angle PCB =..... [2]

(e) X is a point on the line PC so that angle BXC = 60°

Calculate BX

BX=..... cm [3]

13.



The diagram shows four points A. B, C and Don horizontal ground there is a vertical flagpole. FB, held in place by straight wires AF, CF and DF BCD is a straight line, AB = 5.5m, BC = 6.2m and angle FAB = 41°

(a) Show that FB = 4.781 m, correct to 3 decimal places.

[2]

(b) Calculate angle FCB.

Angle FCB=..... [2]

(c) Angle CDF = 186

Show that CD = 8.514, correct to 3 decimal places.

[3]

(d) Angle ABC = 78°.

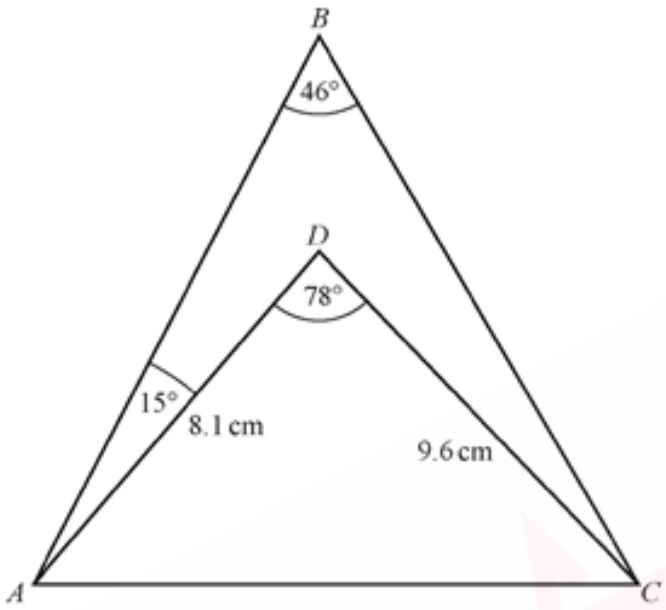
Find AD.

AD =..... m [3]

(e) Find the area of triangle ABD.

..... m^2 [2]

14.



NOT TO SCALE

ABC and ADC are triangles.

4D = 8.1 cm and CD= 9.6cm.

Angle ABC = 46°, angle ADC = 78° and angle BAD = 15^0

(a) Find AC

AC = ...cm [3]

(b) Show that angle DAC = 57°, correct to the nearest degree.

[3]

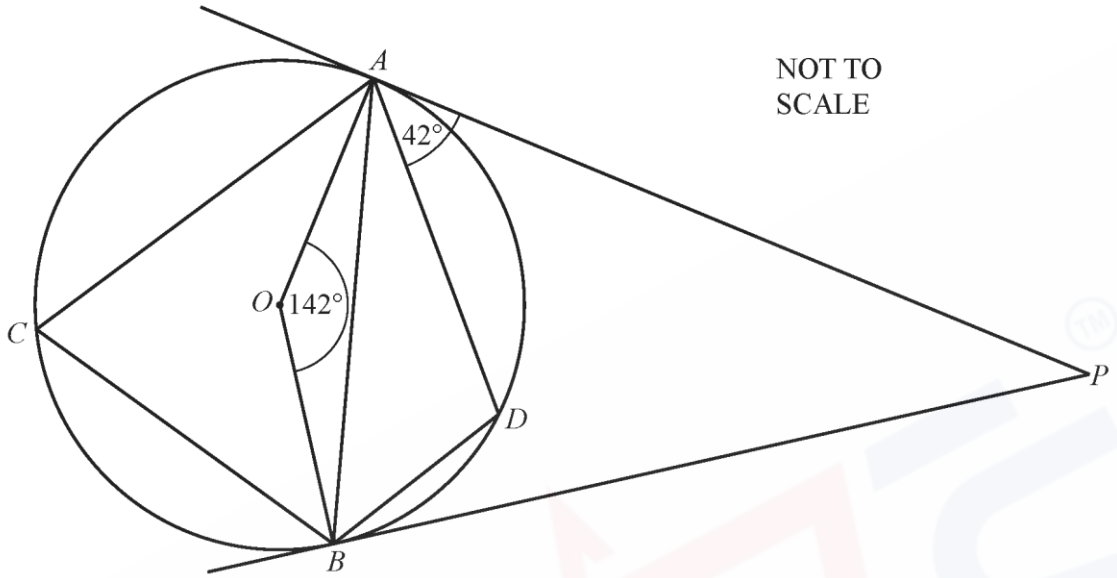
(c) Find BC.

BC = ...cm [3]

(d) Find the area of quadrilateral ABCD.

..... cm^2 [4]

15



A, D, B and C lie on a circle, centre O.
AP is a tangent to the circle at A and BP is a tangent to the circle at B.
Angle $AOB = 142°$ and angle $DAP = 42°$.

(a) Find the value of

(i) angle ABD,

Angle ABD = ... [1]

(ii) angle ACB,

Angle ACB = ... [1]

(iii) angle ADB,

Angle ADB = ... [1]

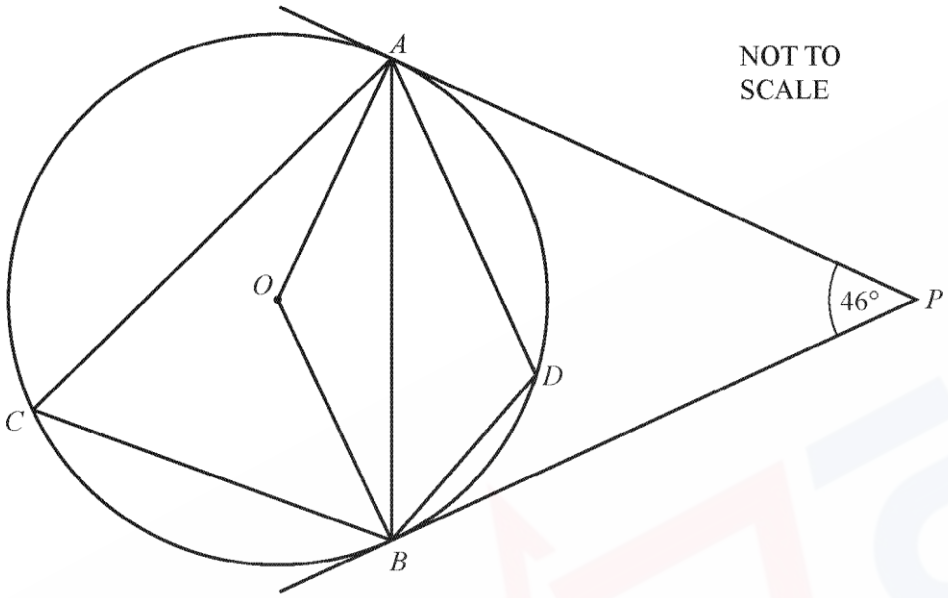
(iv) angle BAD,

Angle BAD = ... [1]

(v) angle APB.

Angle APB = ... [1]

16



A, B, C and D lie on a circle, centre O.
AP and BP are tangents to the circle.
Angle $APB = 46°$.

(a) Complete the statement.

Angle $OAP = 90°$ because .....

..... [1]

(b) Find the value of

(i) angle AOB,

Angle AOB = ... [2]

(ii) angle OAB,

Angle OAB = ... [2]

(iii) angle ACB,

Angle ACB = ... [2]

(iv) angle ADB.

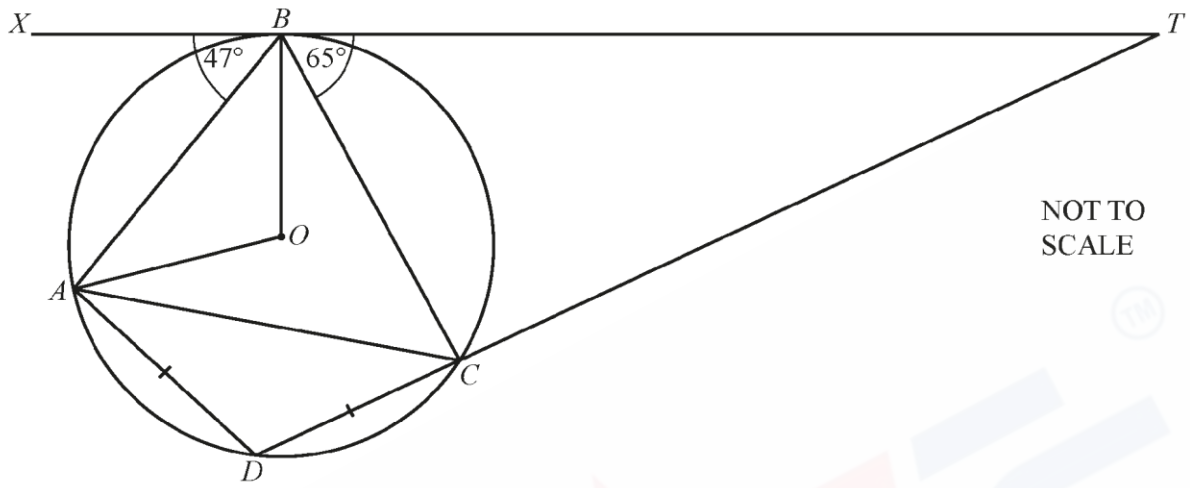
Angle ADB = ... [2]

(c) OB bisects angle ABC.

Find angle OAC.

Angle OAC = ... [3]

17



A, B, C and D lie on a circle, centre O.
$AD = CD$ and XBT is a tangent to the circle at B.
TCD is a straight line.
Angle $XBA = 47°$ and angle $TBC = 65°$.

Find the value of

(a) angle OBX,

Angle OBX = ... [1]

(b) angle AOB,

Angle AOB = ... [2]

(c) angle CAO,

Angle CAO = ... [2]

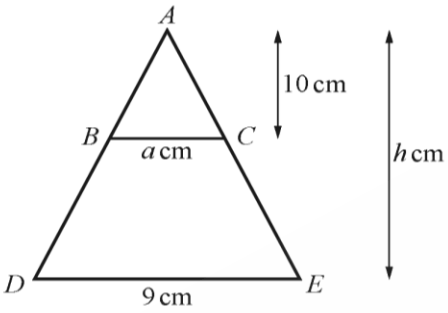
18

(a) A pyramid has a square base with sides of length 9 cm and vertical height h cm.

Find an expression, in terms of h, for the volume of the pyramid.

..... cm^3 [1]

(b)



NOT TO SCALE

ADE is an isosceles triangle.
BC is parallel to DE, $BC = a$ cm and $DE = 9$ cm.
The vertical height of triangle ADE is h cm and the vertical height of triangle ABC is 10 cm.

Show that $a = \frac{90}{h}$

[1]

(c) A square-based pyramid with base of side 9 cm and vertical height h cm contains some water.

When the pyramid is placed on level ground the surface of the water is 10 cm below the vertex of the pyramid (see Diagram 1).

When the pyramid stands vertically on its vertex, the surface of the water is 1 cm below the base of the pyramid (see Diagram 2).

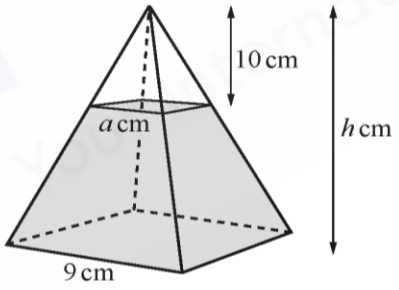


Diagram 1

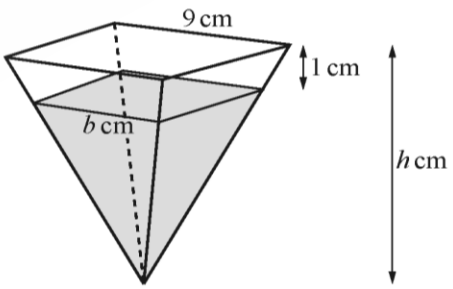


Diagram 2

(i) Use Diagram 1 to find an expression, in terms of a and h, for the volume of the water.

..... cm^3 [1]

(ii) Use Diagram 2 to find an expression, in terms of b and h, for the volume of the water.

..... cm^3 [1]

(iii) Show that $h^3 - 1000 = (h-1)^3$.

[3]

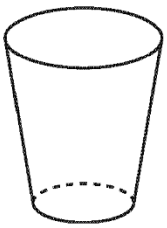
(iv) The equation $h^3 - 1000 = (h-1)^3$ simplifies to $h^2 - h - 333 = 0$.

Use a graphical method to find the value of h.

$h =$ ..... [2]

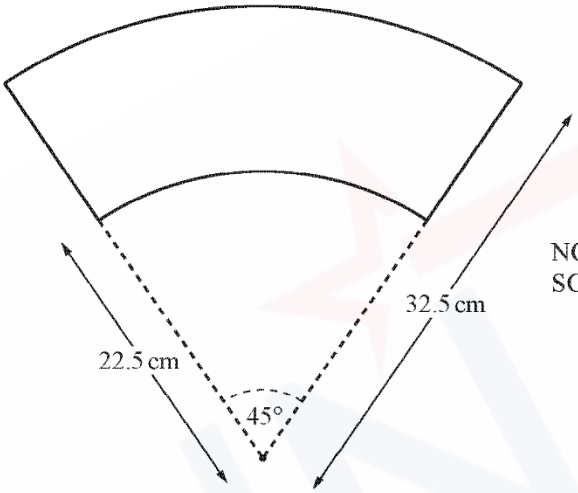
19

The diagram shows a paper cup.



NOT TO SCALE

The curved surface of the cup is made from a sector of a circle with a smaller sector cut from it, as shown below.



NOT TO SCALE

The small sector has radius 22.5 cm and the large sector has radius 32.5 cm.
The sectors have the same centre and both have sector angle 45°.

(a) Show that the radius of the base of the cup is 2.81 cm, correct to 2 decimal places.

[3]

(b) Find the total area of the paper that makes the cup, including the circular base.

..... cm^2 [5]

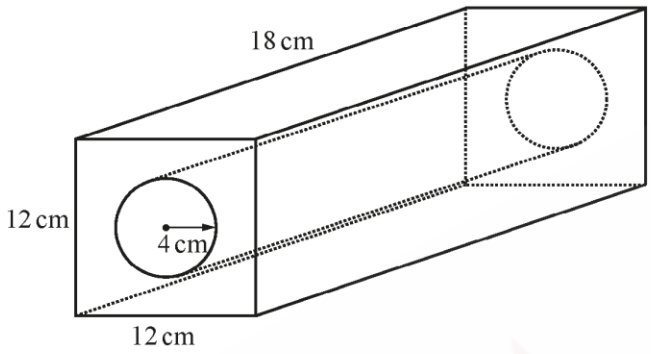
(c) A mathematically similar cup holds 8 times as much liquid as this cup.

Find the total area of the paper that makes the larger cup.

..... cm^2 [2]

20

A piece of metal is in the shape of a cuboid.
The cuboid has length 18 cm, width 12 cm and height 12 cm.
A cylinder is removed from the cuboid.
The cylinder has length 18 cm and radius 4 cm.



NOT TO SCALE

(a) (i) Find the volume of the metal remaining after the cylinder has been removed.

..... cm^3 [3]

(ii) Write your answer to **part (i)** in standard form.

..... cm^3 [1]

(b) Find the total surface area of the metal remaining after the cylinder has been removed.

..... cm^2 [4]

(c) The **cylinder removed** is melted and formed into 16 identical spheres.

(i) Calculate the volume of **one** sphere.

..... cm^3 [1]

(ii) Calculate the radius of one sphere.

..... cm [2]