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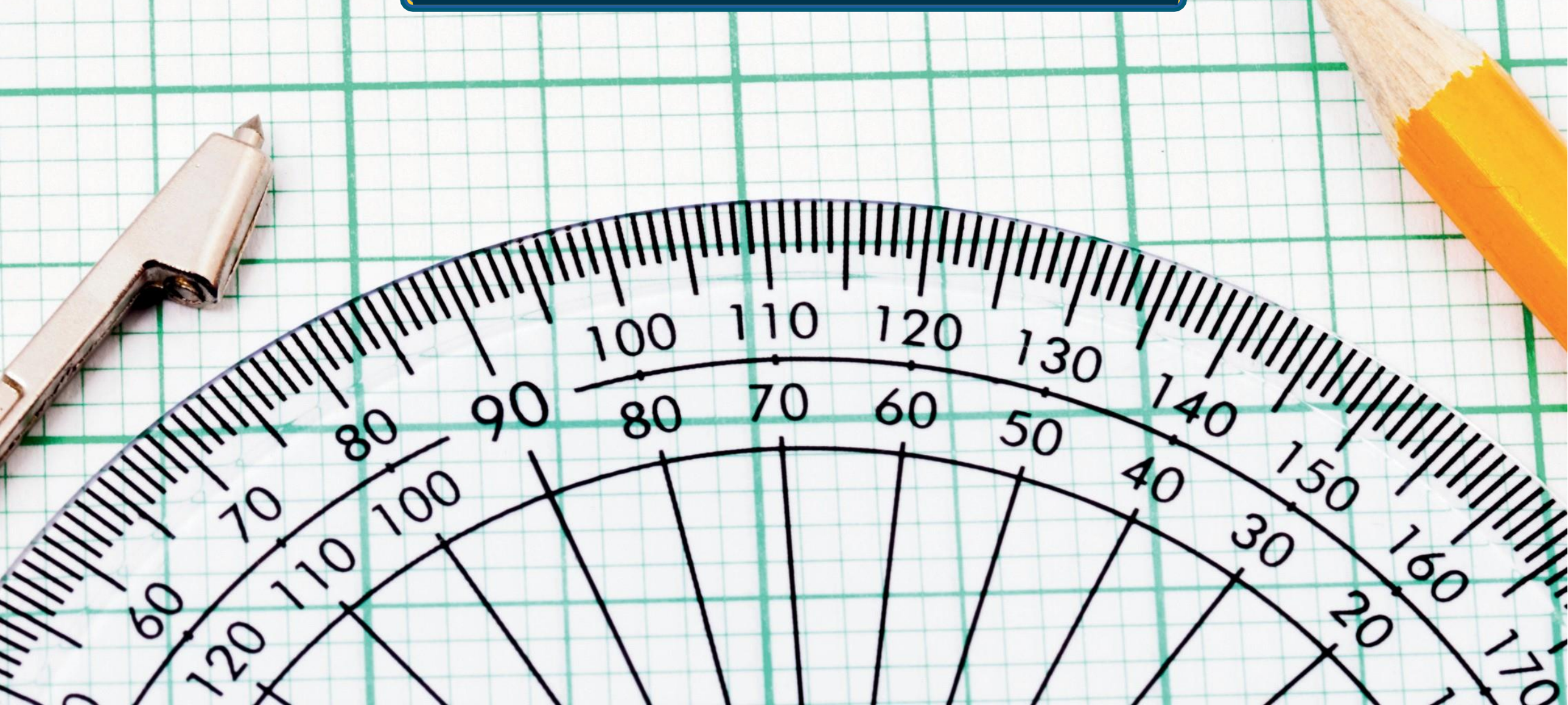
MATH TOPICAL WORKSHEETS

Numbers and Algebra



1

As p and $(q+2)^2$ are directly proportional, a formula for p can be found by multiplying a constant (of proportionality) by $(q+2)^2$.

$$p = k(q+2)^2$$

[1]

Substitute $p = 1$ and $q = 1$ into the formula to form an equation.

$$1 = k \times (1+2)^2$$
$$1 = 9k$$

Solve the equation to find the value of k.

$$9k = 1$$
$$k = \frac{1}{9}$$

Substitute the value of k back into the formula for p.

$$p = \frac{1}{9}(q+2)^2$$

Substitute $q = 10$ into the formula to find the value of p.

$$p = \frac{1}{9} \times (10+2)^2$$

[1]

$p = 16$ [1]

2

Look to cancel factors on the numerator and denominator before multiplying.
Starting with the numbers, we can see that 3 is a multiple of 9 and 3, and that 5 is a multiple of 5 and 20.
Looking at letters, we can see there is an x on the numerator that will cancel with the x on the denominator.

$$\frac{\cancel{3}^{1}\cancel{x}}{\cancel{5}\cancel{x}_{1}} \times \frac{\cancel{9}^{3}\cancel{x}}{\cancel{20}_{4}} = \frac{1\times 3}{1\times 4} = \frac{3}{4}$$

$\frac{3}{4}$

Equivalent fraction [1]
Fully simplified [1]

3

First simplify $\sqrt{8}$.

$$\sqrt{8} = \sqrt{4}\times\sqrt{2} = 2\sqrt{2}$$

You can now cancel out the common factor of 2.

$$\frac{2\sqrt{2}}{2\sqrt{2}-2} = \frac{\sqrt{2}}{\sqrt{2}-1}$$

[1]

Rationalise the denominator by multiplying the top and bottom by $\sqrt{2}+1$.

$$\frac{\sqrt{2}}{(\sqrt{2}-1)} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)} = \frac{\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

[1]

Expand and simplify the expressions.

$$\frac{2+\sqrt{2}}{(\sqrt{2})^2+\sqrt{2}-\sqrt{2}-1} = \frac{2+\sqrt{2}}{2-1} = \frac{2+\sqrt{2}}{1}$$

$2+\sqrt{2}$ [1]

4

To find the inverse, let $y = 5^x$ and swap the x and y.

$$x = 5^y$$

Let $y = 2$.

$$x = 5^2$$

$$x = 25 \text{ [1]}$$

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5

The question describes an increase of 5%, so add 5% to the original 100% and divide by 100 to find the percentage multiplier.

$$(100 + 5) \div 100 = 1.05$$

i) If you know the original amount and you wanted to find the final amount after 2 years, you would multiply the original amount by the percentage multiplier squared.

$$\text{original amount} \times 1.05^2 = 882$$

[1]

For this reverse percentages question you need to work backwards.
Divide the final amount by the percentage multiplier squared to find the original amount.

$$\text{original amount} = 882 \div 1.05^2$$

$800 [1]

As the answer is an exact number of dollars, you don't need to include the 2 d.p.

ii) Substitute different integer (whole number) values for n into the equation 882×1.05^n, until the final amount is greater than $1100.

E.g. $n = 4$

$$882 \times 1.05^4 = 1072.07651...$$

[1]

This answer is just a little bit lower than needed, so try a higher value for n.

E.g. $n = 5$

$$882 \times 1.05^5 = 1125.68033...$$

This has just exceeded the value of $1100.

5 years [1]

6

Remove the fraction (by multiplying both sides by $y + 2$)

$$t(y + 2) = 2 - 3y$$

[1]

Expand the brackets (by multiplying the terms inside by t)

$$ty + 2t = 2 - 3y$$

Collect the y terms on one side (for example, by adding $3y$ to both sides then subtracting $2t$ from both sides)

$$ty + 2t + 3y = 2$$
$$ty + 3y = 2 - 2t$$

[1]

Factorise y out of the terms on the left

$$y(t + 3) = 2 - 2t$$

[1]

Get y on its own (by dividing both sides by $t + 3$)

$$y = \frac{2 - 2t}{t + 3}$$ [1]

$y = \frac{2t - 2}{-t - 3}$ is also accepted

7

Find 62 out of 80 as a percentage

$$\frac{62}{80} \times 100 = 77.5\%$$

[1]

This is in the middle row of the table

Merit [1]

8

To find an average speed, we need to know the total distance and the total time

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

For each bullet point in the question, we will find the total time and the total distance covered, making use of $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$ or

$\text{Distance} = \text{Speed} \times \text{Time}$

We also need to find any wait times between journeys, as these also contribute to the total time

Home to airport

To find the distance, we need to use $\text{Distance} = \text{Speed} \times \text{Time}$

$$55 \text{ minutes} = \frac{55}{60} \text{ of an hour or } \frac{11}{12} \text{ hrs}$$

$$\text{Distance} = 18 \times \frac{11}{12} = 16.5 \text{ km}$$

Speed × Time [1]
16.5 [1]

Wait time until flight

Left home at 16 30 and took 55 minutes, so arrived at airport at 17 25. Flight left New York at 22 15

$$\text{Wait time of 17 25 to 22 15} = 4\text{h } 50\text{m} = 4\frac{50}{60} \text{ hrs} = \frac{29}{6} \text{ hrs}$$

Flight from New York to Geneva

To find the arc length we can use the formula $\frac{\theta}{360} \times 2\pi r$ (this is the fraction of the circle, in degrees, multiplied by the formula for the circumference of a circle) where θ is the angle in degrees, and r is the radius

$$\frac{55.5}{360} \times 2\pi \times 6400 = \frac{5920\pi}{3} = 6199.409503... \text{ km}$$

55.5 ÷ 360 [1]
Full calculation [1]
6200 or 6199 to 6200 [1]

To find the time; the flight left at 22 15 (New York time) and arrived at 11 25 Geneva time. Geneva is 6 hours ahead of New York.

11 25 Geneva time is 05 25 New York time

$$22\,15 \text{ to } 05\,25 \text{ is } 7\text{h } 10\text{m} = 7\frac{10}{60} = \frac{43}{6} \text{ hrs}$$

Wait time until bus

The flight arrives at 11 25 (Geneva time) but the bus journey only starts at 13 00, so there is a wait time

$$11\,25 \text{ to } 13\,00 \text{ is } 1\text{h}35\text{m} = 1\frac{35}{60} = \frac{19}{12} \text{ hrs}$$

Bus Journey from Geneva to Chamonix

To find the distance, we need to use $\text{Distance} = \text{Speed} \times \text{Time}$

$$1\text{h}36\text{m} = 1 + \frac{36}{60} = 1.6 \text{ hrs}$$

$$\text{Distance} = 65 \times 1.6 = 104 \text{ km}$$

Speed × Time [1]
104 [1]

Average speed for the whole journey

Using $\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$ and the distances and times found in km and hours, plus the wait times in hours Use the most accurate, unrounded, values you can

$$\text{Average Speed} = \frac{16.5 + \frac{5920\pi}{3} + 104}{\frac{11}{12} + \frac{29}{6} + \frac{43}{6} + \frac{19}{12} + 1.6} = 392.5409629\ldots$$

Correct total time of 16.1 hours [2]

393 km / h [2]
392 to 393 allowed

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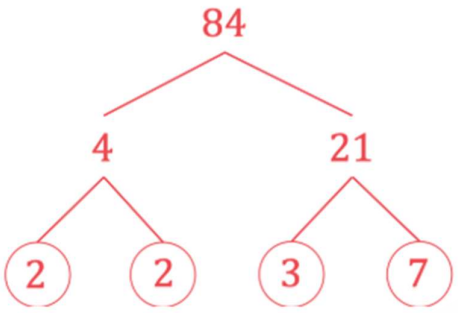
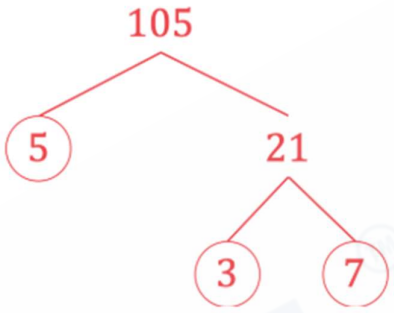
9

Break down each number by repeatedly finding factor pairs until all factors are prime numbers.





Write each number as a product of its prime factors.

$$84 = 2^2 \times 3 \times 7$$
$$105 = 3 \times 5 \times 7$$

For either number written as a product of its primes [1]

Find all the factors that are common to both numbers and multiply them together to find the HCF.

$$3 \times 7$$

21 [1]

10

Find all factors that occurs in both terms of the expression.

Common factor: 3

Take the common factors outside a pair of single brackets and find the terms inside the brackets by working out what you would need to multiply the common term by to give you the original term.

$3(4x + 5)$ [1]

11

1st sequence: Add the two previous terms to work out the next term.

2 4 6 10 16

[1]

2nd and 3rd sequences: Use trial and improvement to find the missing terms.

1 3 4 7 11

[1]

2 −1 1 0 1

[1]

12

Method 1

Cancel the fraction

$$\frac{w^2}{w^3} = \frac{\cancel{w} \times \cancel{w}}{w \times \cancel{w} \times \cancel{w}} = \frac{1}{w}$$

$\frac{1}{w}$ [1]

Method 2

Use the index law $a^m \div a^n = a^{m-n}$

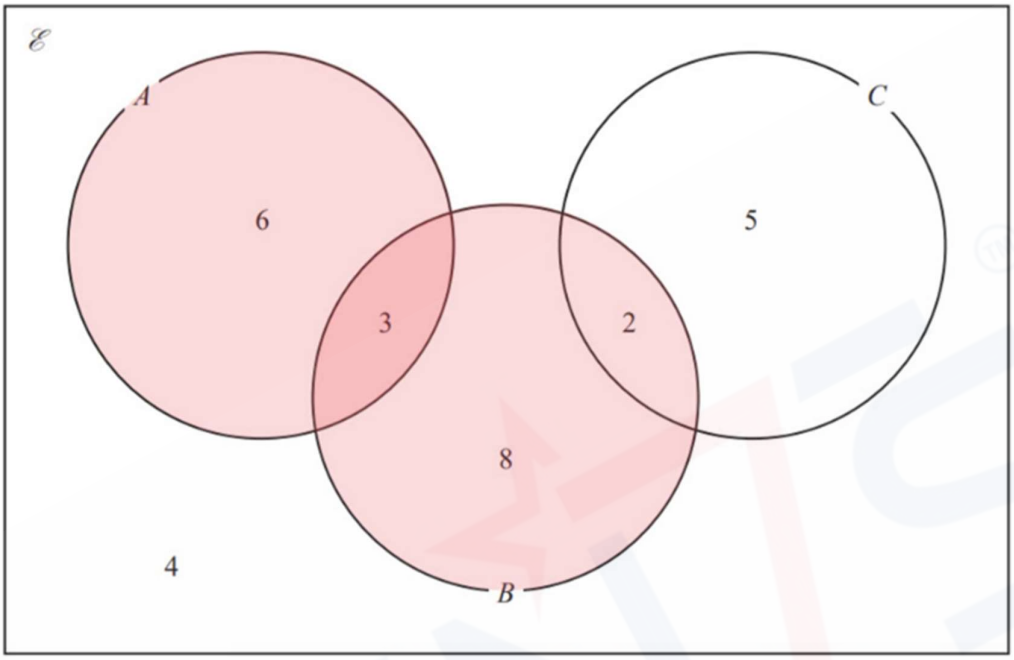
$$w^2 \div w^3 = w^{2-3} = w^{-1}$$

w^{-1} [1]

Both answers in method 1 and method 2 are accepted

13A

$n(A \cup B)$ means the number of elements in A **or** B. It is everything shaded below.

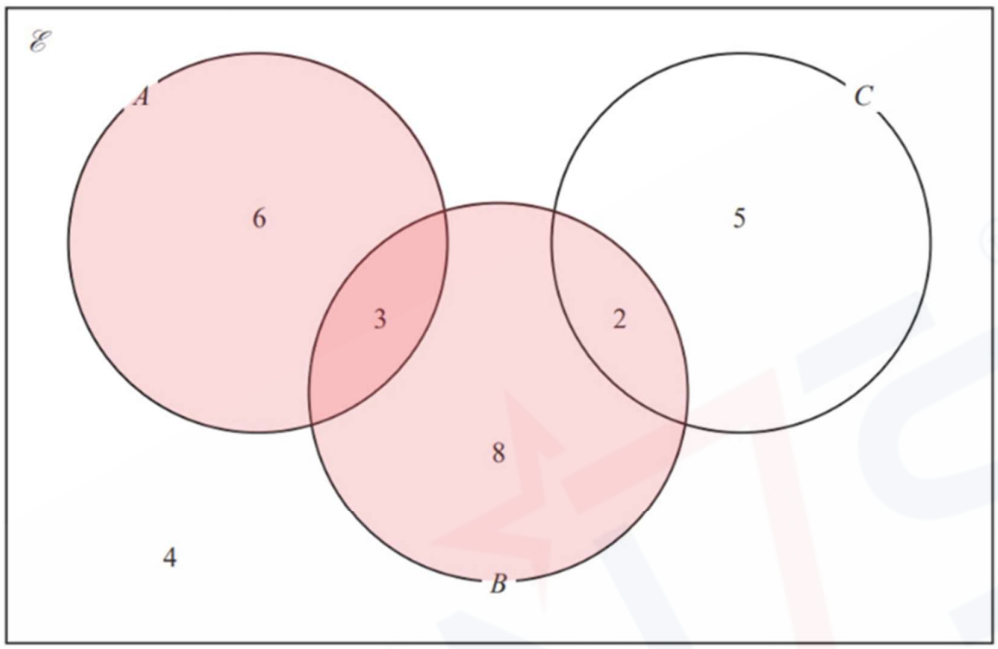


$$n(A \cup B) = 6 + 3 + 8 + 2$$

Answer = 19 [1]

13B

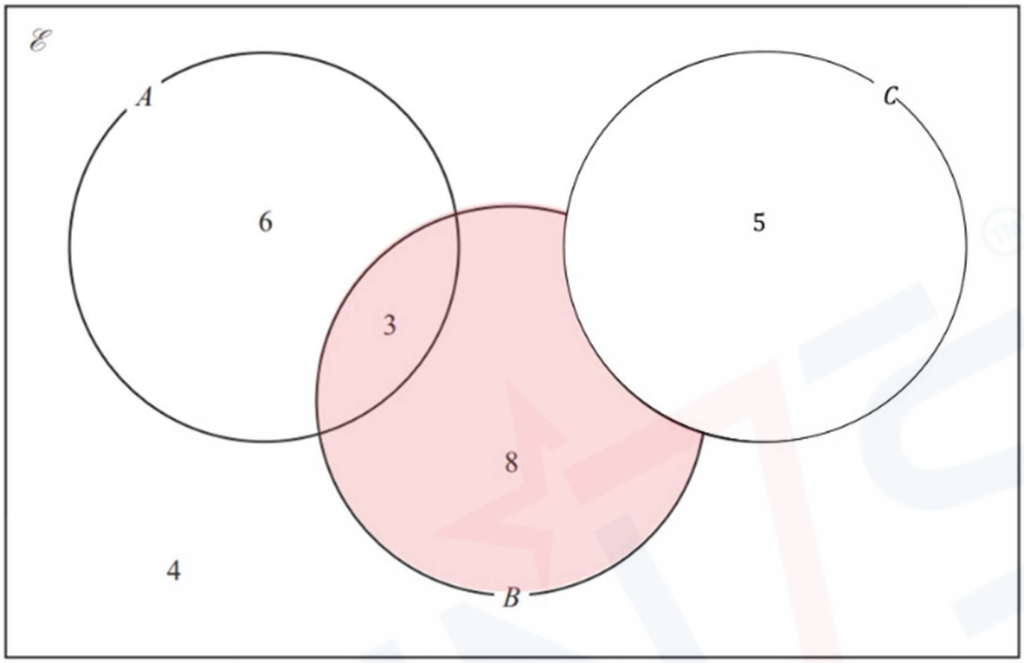
$n(A \cap B)$ means the number of elements in A **and** B. It is the overlapping shaded part below.



Answer = 3 [1]

13C

$n(B \cap C')$ means the number of elements in B **and not** in C. It is everything shaded below.



$$n(B \cap C') = 3 + 8$$

Answer = 11 [1]

13D

$n(A' \cup B' \cup C')$ means the number of elements that are either not in A or not in B or not in C. None of the elements are in A, B and C at the same time, so all of the elements are either not in A or not in B or not in C.

Add all the numbers in the diagram.

$$n(A' \cup B' \cup C') = 6 + 3 + 8 + 2 + 5 + 4$$

Answer = 28 [1]

14

Find the total fees charged for tents by multiplying the number of tents by the cost per tent.

$$54 \times \$15 = \$810$$

Find the total fees charged for caravans by multiplying the number of caravans by the cost per caravan.

$$18 \times \$25 = \$450$$

Either calculation [1]

Find the total fees charged by adding together the total fees charged for tents and caravans.

$$\$810 + \$450$$

[1]

$1 260 [1]

15

Find a common denominator and rewrite both fractions We can use 12, as 4 and 6 are both factors of 12

$$\frac{3}{4} + \frac{1}{6} = \frac{3 \times 3}{4 \times 3} + \frac{1 \times 2}{6 \times 2} = \frac{9}{12} + \frac{2}{12}$$

Common denominator, could also be written as $\frac{18}{24} + \frac{4}{24}$ *for example* [1]

We can now carry out the addition

$$\frac{9}{12} + \frac{2}{12} = \frac{11}{12}$$

This is already in its simplest form as 11 and 12 have no common factors

$\frac{11}{12}$ [1]

16

Perform the calculation using your calculator.
Use the fraction button on your calculator and use the arrow keys to move around the calculator screen to make it look just like the given calculation.

$$\frac{16.379 - 0.879}{4.2} \times 1.241 = 4.579880...$$

[1]

Round your answer to 2 s.f.

4.6 [1]

17

The values of a and b can be found by substituting in total numbers of dots from the diagram, then solving the equations

The first diagram has 1 dot, so when $n = 1$, $T = 1$
Substitute this into the equation and call this equation (1)

$$n = 1, T = 1$$
$$1 = a(1)^3 + b(1)^2$$
$$1 = a + b$$

[1]

The first two diagrams have a total of $1 + 5 = 6$ dots, so when $n = 2$, $T = 6$
Substitute this into the equation and call this equation (2)

$$n = 2, T = 6$$
$$6 = a(2)^3 + b(2)^2$$
$$6 = 8a + 4b$$

[1]

Simplify the second equation by dividing by the common factor of 2

$$3 = 4a + 2b$$

Write the two equations out next to each other and number them clearly

$$a + b = 1 \quad (1)$$
$$4a + 2b = 3 \quad (2)$$

Make the b terms equal by multiplying all parts of equation (1) by 2

$$2a + 2b = 2 \quad (3)$$
$$4a + 2b = 3 \quad (2)$$

Eliminate the b terms by subtracting equation (3) from equation (2). Be careful to subtract every term.

$$\begin{array}{rll} 4a + 2b = 3 & (2) \\ -\ (2a + 2b = 2) & (3) \\ \hline 2a \quad\quad = 1 \end{array}$$

[1]

Solve the equation to find a by dividing both sides by 2.

$$a = \frac{1}{2}$$

[1]

Substitute $a = \frac{1}{2}$ into either of the two original equations.

$$(1) \quad a + b = 1$$
$$\left(\frac{1}{2}\right) + b = 1$$

Solve this equation to find b.

$$b = 1 - \frac{1}{2}$$
$$= \frac{1}{2}$$

$a = \frac{1}{2}$, $b = \frac{1}{2}$ [1]

18

i) Remember that numbers in standard form are written in the form $a \times 10^n$ where $1 \leq a < 10$.

To go from 7.2 to 72 000 we would need to multiply by 10 000 (or 10^4)

7.2×10^4 [1]

ii) Remember that numbers in standard form are written in the form $a \times 10^n$ where $1 \leq a < 10$.

To go from 1.8 to 0.0018 we would need to divide by 1000 (or 10^3) Because we are dividing rather than multiplying, the power of 10 will be negative

1.8×10^{-3} [1]

19

Calculate how much the original amount has increased by.

$$77 - 63 = 14$$

[1]

Divide the increase by the original amount.

$$\frac{14}{63} = \frac{2}{9}$$

[1]

Multiply by 100 to write as a percentage.

$$\frac{2}{9} \times 100 = 22\frac{2}{9} = 22.22...$$

$22\frac{2}{9}\%$ [1]

Rounding to 3sf is also acceptable: 22.2%

20

Multiply both sides by 5, remember to apply this to every term

$$15(x-4)+(x+2)=30$$

[1]

Expand the bracket on the left

$$15x-60+(x+2)=30$$

Simplify the left

$$16x-58=30$$

[1]

Add 58 to both sides

$$16x=88$$

[1]

Divide both sides by 16

$$x=\frac{88}{16}=\frac{11}{2}=5.5$$

$x=5.5$ [1]

Or as a fraction $\frac{11}{2}$

21

Find the fraction of the price that gives profit (by splitting the ratio $5:3$ into the fractions $\frac{5}{5+3}$ and $\frac{3}{5+3}$)

fraction of price that gives profit is $\frac{3}{8}$

[1]

Find this fraction of \$6 (by multiplying 6 by $\frac{3}{8}$)

$$6\times\frac{3}{8}=\frac{6\times3}{8}=\frac{18}{8}=\frac{9}{4}=2\frac{1}{4}=2.25$$

Give the answer as a price in dollars (2 decimal places)

\$2.25 [1]

22a

The area of a triangle is $\frac{1}{2}\times$ base $\times$ height Substitute the base as $(2x+5)$ and the height as $4(x-1)$ into the formula

$$\frac{1}{2}\times(2x+5)\times4(x-1)$$

[1]

Simplify the expression (using $\frac{1}{2}\times4=2$)

$$2(2x+5)(x-1)$$

Form an equation in x (by setting the expression to equal 30)

$$2(2x+5)(x-1)=30$$

Divide both sides by 2

$$(2x+5)(x-1)=15$$

Divide both sides by 2

$$(2x+5)(x-1)=15$$

Expand the brackets

$$2x^2-2x+5x-5=15$$
$$2x^2+3x-5=15$$

[1]

Form a quadratic equation (by subtracting 15 from both sides to get = 0)

$$2x^2+3x-5-15=0$$
$$2x^2+3x-20=0$$

$2x^2+3x-20=0$ [1]

Answer correctly obtained with no errors or omissions

22b

"Use factorisation" means write the quadratic as two brackets multiplied together
Multiply the first and last numbers together

$$2\times -20=-40$$

Find two numbers that multiply to give -40 and add to give the middle number, 3

8 and -5

Split the middle term of the quadratic into two parts with these numbers

$$2x^2+8x-5x-20$$

Factorise fully the first pair of terms and factorise fully the second pair of terms

$$2x(x+4)-5(x+4)$$

[1]

Factorise out the common bracket of $(x+4)$

$$(x+4)(2x-5)$$

[1]

Write out the full equation

$$(x+4)(2x-5)=0$$

Solve the first bracket equal to zero and the second bracket equal to zero

$$x+4=0 \quad \text{or} \quad 2x-5=0$$
$$x=-4 \qquad\qquad 2x=5$$
$$x=\frac{5}{2}$$

$x=-4$ or $x=\frac{5}{2}$ [1]

22c

Find the length of side AB by substituting in x from part (b)

$AB=4(x-1)$ cannot have $x=-4$ otherwise AB is negative must use $x=\frac{5}{2}$ $AB=4\left(\frac{5}{2}-1\right)=6$

[1]

Find the length of side AC (by substituting in $x=\frac{5}{2}$)

$$AC=\left(2\times\frac{5}{2}+5\right)=10$$

Find BC by Pythagoras' Theorem

$$BC=\sqrt{6^2+10^2}$$

[1]

11.7 cm [1]

23

The result required is in completed square form.
To complete the square, set up a squared bracket with two terms; x and half of the coefficient of the x term - in this case 5. The squared brackets would then produce an extra, unwanted constant term, so we need to subtract this; in this case the unwanted constant would be $5^2 = 25$.

$$x^2 + 10x + 14 = (x + 5)^2 - 25 + 14$$

[1]

$$= (x + 5)^2 - 11$$

$x^2 + 10x + 14 = (x + 5)^2 - 11$ [1]

24

Substitute "-2" for "y" in the (right-hand side of the) formula.

If using your calculator it is good practice to put negative numbers in brackets.

$$w = 5(-2)^2 - (-2)^3$$

[1]

$$\begin{aligned} w &= 5 \times 4 - (-2)^3 \\ &= 20 - (-8) \\ &= 20 + 8 \\ &= 28 \end{aligned}$$

$w = 28$ [1]

25

i) Multiples of 8 are numbers that are in the 8 times table. $4 \times 8 = 32$.

The multiple of 8 is 32 [1]

ii) Square numbers are numbers that can be made by multiplying an integer by itself. $6^2 = 6 \times 6 = 36$.

The square number is 36 [1]

iii) Prime numbers are positive integers that have exactly two factors: 1 and itself. Ignore the numbers that have other factors.

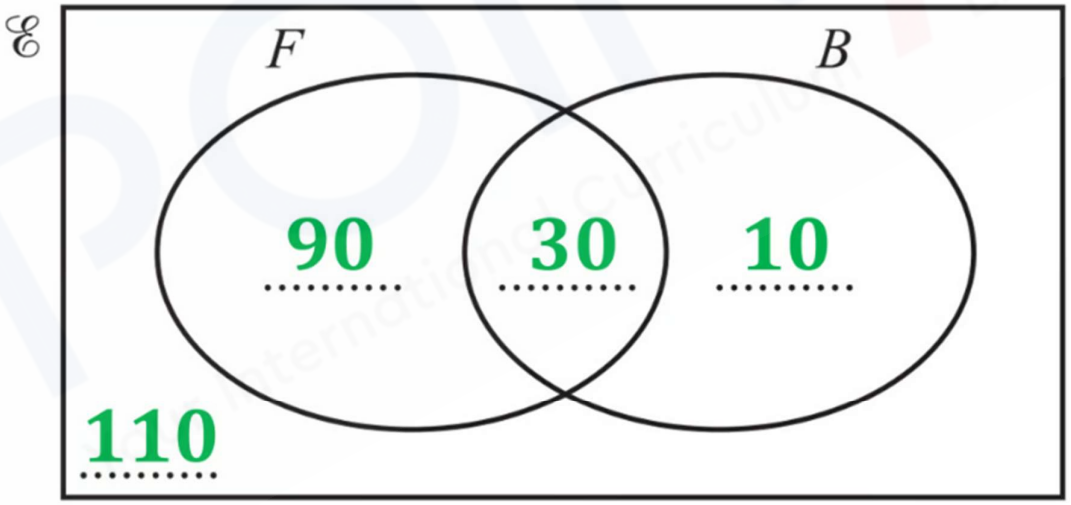
32, 34, 36, 38 all have 2 as a factor.
33, 36, 39 all have 3 as a factor.
35 has 5 as a factor.

The prime number is 37 [1]

26a

$240 - 90 - 10 - 30 = 110$

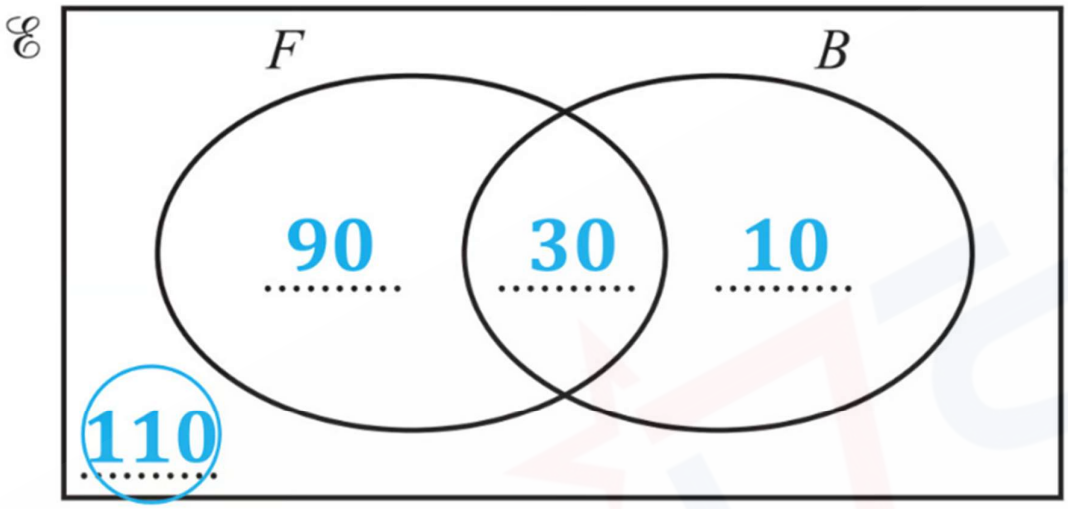
Any one correct value [1]



All values correct [1]

26b

$n(F' \cap B')$ means "the number of students who do not play football, and, do not play baseball
This is the number outside the two circles



110 [1]

27

Simply put the calculation into your calculator.
On older calculators you will need to pay attention to BIDMAS/ order of operations- 6.4^2 first, then add 2.38, then square root

6.583312236 [1]

Write down all the digits in your calculator display

28

Using the given equation and square rooting both sides

$$\sqrt{8300} = \sqrt{100 \times 83}$$

Using the rule for surds: $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

$$\sqrt{100} \times \sqrt{83}$$

$\sqrt{100} = 10$ and $\sqrt{83}$ is close in value to $\sqrt{81}$ which is 9

$$10 \times 9 \text{ (approximately)}$$

$$= 90$$

So the 2nd option, 90, is the closest in value [1]

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