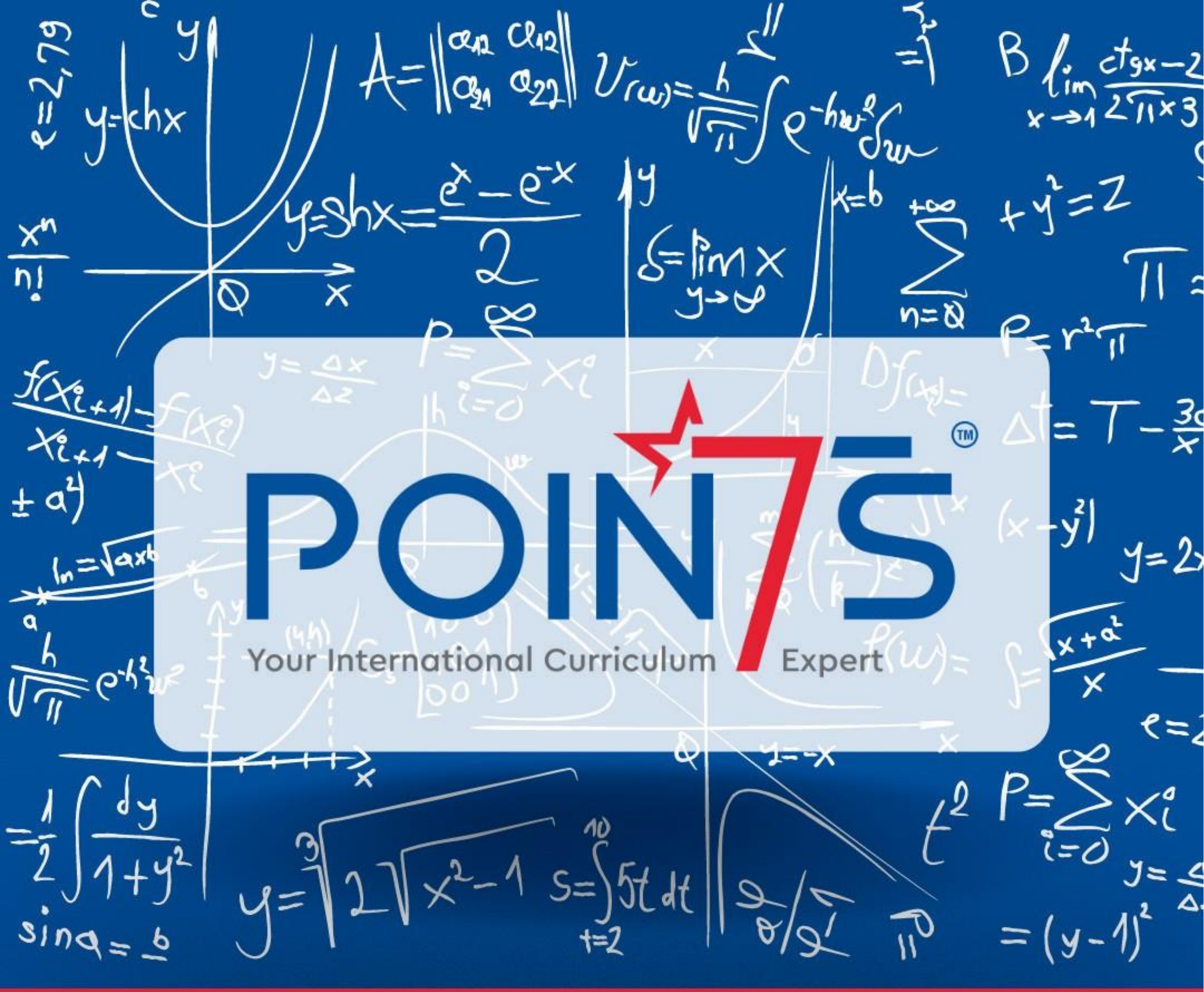

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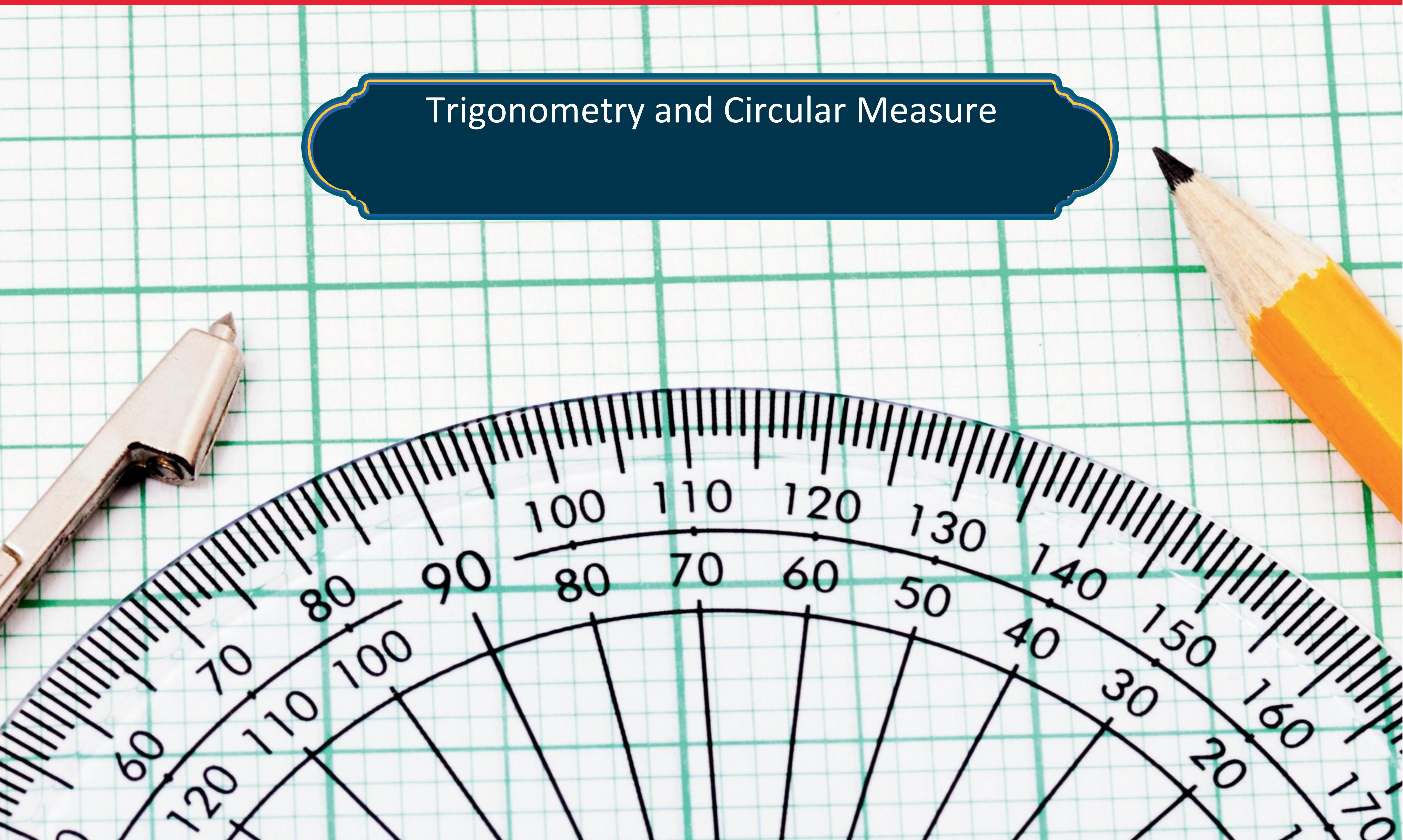
MATH TOPICAL WORKSHEETS

Trigonometry and Circular Measure



MS-WORKSHEET

Topic:	Trigonometry and Circular Measure
Board:	IGCSE
Subject:	Add math

1

Use the trigonometric identity $1+\cot^2 x=\operatorname{cosec}^2 x$ to rewrite the equation

$$3(\operatorname{cosec}^2 x-1)-14\operatorname{cosec}x-2=0$$

[1]

Expand the bracket

$$3\operatorname{cosec}^2 x-3-14\operatorname{cosec}x-2=0$$

$$3\operatorname{cosec}^2 x-14\operatorname{cosec}x-5=0$$

[1]

Factorise the quadratic equation

$$(\operatorname{cosec}x-5)(3\operatorname{cosec}x+1)=0$$

[1]

Solve the first bracket equal to 0

either $\operatorname{cosec}x-5=0$

$$\frac{1}{\sin x}=5$$

$$\sin x=\frac{1}{5}$$

[1]

$$x=\sin^{-1}\left(\frac{1}{5}\right)=11.536...$$

Find other solutions in the range

or $x=(180-11.536...)=168.463...$

Solve the first bracket equal to 0

or $3\operatorname{cosec}x+1=0$

$$\operatorname{cosec}x=-\frac{1}{3}$$

$$\frac{1}{\sin x}=-\frac{1}{3}$$

$\sin x=-3$ which has no solutions

11.5 and 168.5 [1]

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2

The circle has radius = 6cm

Calculate the length of the arc

$$\text{arc length} = 2(6 + 5\pi) - (2 \times 6)$$

$$\text{arc length} = 12 + 10\pi - 12$$

$$\text{arc length} = 10\pi$$

[1]

Calculate the circumference of the circle

$$C = 2 \times \pi \times 6$$

$$C = 12\pi$$

Work out the fraction of the area that we want

$$\frac{10\pi}{12\pi} = \frac{5}{6}$$

[1]

Calculate the area of the sector

$$\text{area of sector} = \frac{5}{6} \times \pi \times 6^2$$

[1]

$$\text{area of sector} = 30\pi$$

30π cm^2 [1]

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3

Start with the left hand side and factorise the difference of two squares

$$\frac{\sin^4 y - \cos^4 y}{\cot y} = \frac{(\sin^2 y - \cos^2 y)(\sin^2 y + \cos^2 y)}{\cot y}$$

[1]

$\cot y \equiv \frac{1}{\tan y}$ so

$$= (\sin^2 y - \cos^2 y)(\sin^2 y + \cos^2 y) \times \tan y$$

[1]

$\sin^2 y + \cos^2 y \equiv 1$ so

$$= (\sin^2 y - \cos^2 y) \times \tan y$$

[1]

$$= ((1 - \cos^2 y) - \cos^2 y) \times \tan y$$

$$= (1 - 2\cos^2 y)\frac{\sin y}{\cos y}$$

Expand the bracket

$$= \frac{\sin y}{\cos y} - 2\sin y \cos y$$

$$= \tan y - 2\sin y \cos y$$

$\tan y - 2\sin y \cos y$ [1]

4a

In radians, Area of sector $= \frac{1}{2}r^2\theta$

Substitute known values into the equation.

$$35 = \frac{1}{2}(10)^2 \times \theta$$

Rearrange and solve for θ.

$$35 = 50 \times \theta$$

$$\theta = \frac{35}{50}$$

$$\theta = 0.7$$

0.7 [1]

4b

Find the arc length CD.

$$CD = r\theta$$
$$CD = 10 \times 0.7$$
$$CD = 7cm$$

[1]

Work out length $\frac{AB}{2}$ by using trigonometry.

$$\sin(0.35) = \frac{AB/2}{12}$$
$$\frac{AB}{2} = 4.11477$$

[1]

Double to find AB.

$$AB = 8.229547cm$$

[1]

Add the sides to find the perimeter.

$$\text{Perimeter} = 7 + 8.23 + 2 + 2$$

19.2 cm [1]

4c

Area of shaded region = Area of triangle – Area of sector

We are told the area of the sector is $35cm^2$.

Find the area of the triangle AOB.

$$\text{Area} = \frac{1}{2}ab \sin C$$
$$\text{Area} = \frac{1}{2}(12)^2 \times \sin(0.7)$$

[1]

$$\text{Area} = 46.38$$

[1]

Subtract the area of the sector from the area of the triangle.

$$46.38 - 35 = 11.38$$

11.4 cm² [1]

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5a

Since the graph passes through $(0, \ -4)$, we know the the graph of $\tan x$ has been translated 4 units down.

Therefore,

$a = -4$ [1]

The period of $y = \tan x$ is $180°$. Since the graph of $y = a + 2\tan bx$ has a period of $480°$,

$$\frac{180}{b} = 480$$

[1]

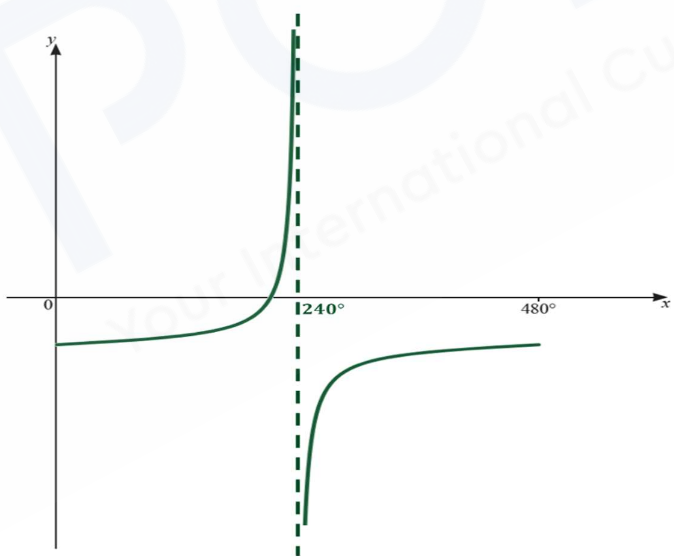
Solve to find b.

$$b = \frac{180}{480}$$

$b = \frac{3}{8}$ [1]

5b

The graph passes through the point (0, -4) so use this as a starting point. The graph will be in the shape of a tan graph with a period of 480°, meaning it will have an asymptote at $\frac{480}{2} = 240°$.



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correct shape [1]

correct asymptote [1]

6

i) Looking only at the left-hand side of the equation, and using the reciprocal trig function $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$, we have

$$\frac{1}{\left(1 + \frac{1}{\sin\theta}\right)(\sin\theta - \sin^2\theta)}$$

[1]

Expanding the brackets gives

$$\frac{1}{\sin\theta - \sin^2\theta + 1 - \sin\theta}$$

[1]

Simplifying gives

$$\frac{1}{1 - \sin^2\theta}$$

Using the identity $1 - \cos^2\theta \equiv \sin^2\theta$

$$\frac{1}{\cos^2\theta}$$

We know that $\frac{1}{\cos\theta} = \sec\theta$, therefore

$$\frac{1}{\cos^2\theta} = \sec^2\theta$$

$\sec^2\theta$ as required [1]

ii) Using part (i), and the reciprocal trig function $\frac{1}{\cos^2\theta} = \sec^2\theta$, we have

$$\cos^2\theta = \frac{3}{4}$$

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Finding the square root of both sides

$$\cos\theta = \frac{\pm\sqrt{3}}{2}$$

[1]

Using a calculator to find the first solutions, we find that

$$\theta = 30°, 150°$$

[1]

Using a CAST diagram, given that the required interval is $-180° \leq \theta \leq 180°$, we can find the other solutions

$\theta = -150°, -30°, 30°, 150°$ [1]

7

The amplitude of the Sine graph is $|a|$

Therefore:

$a = 20$ [1]

The period of the Sine graph is $\frac{360°}{b}$ because bx is a horizontal stretch of scale factor $\frac{1}{b}$

Therefore:

$$\frac{360°}{b} = 180°$$

$b = 2$ [1]

We now know that $a = 20$ and $b = 20$. The graph of $y = 20\sin(2x)$ will cross the x axis at $(90°, 0)$.

Since we are told the graph passes through $(90°, -3)$, we know that the graph has been translated vertically downwards by three units.

Therefore:

$c = -3$ [1]

8

i) Split the expression and factorise each part.

$$6xy + 3y = 3y(2x+1)$$

$$4x + 2 = 2(2x+1)$$

Combine.

$$3y(2x+1) + 2(2x+1)$$

Simplify.

$(3y+2)(2x+1)$ [1]

ii) Substitute $x = \sin(\theta)$ and $y = \cos(\theta)$ into the answer from part (i).

$$(3\cos(\theta)+2)(2\sin(\theta)+1) = 0$$

Solve $3\cos(\theta) + 2 = 0$.

$$3\cos(\theta) = -2$$

$$\theta = \cos^{-1}\left(-\frac{2}{3}\right)$$

$$\theta = 131.81°$$

[1]

By using the CAST method, or by examining the Cosine curve, work out other solutions in the range $0° < \theta < 360°$

$$360 - 131.81$$

$$\theta = 228.19°$$

[1]

Solve $2\sin(\theta) + 1 = 0$.

$$2\sin(\theta) = -1$$

$$\theta = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = -30°$$

[1]

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By using the CAST method, or by examining the Sine curve, work out other solutions in the range $0° < \theta < 360°$

$$180 + 30$$

$$\theta = 210°$$

$$360 - 30$$

$$\theta = 330°$$

Give all solutions in the range.

$\theta = 131.8°$, $\theta = 228.2°$, $\theta = 210°$, $\theta = 330°$ [1]

9

The coefficient of cos will apply a stretch to $y = \cos(x)$, scale factor a, parallel to the y-axis

Compare the height of the cos graph to the graph in the question and notice that it is 3 times larger

$$a = 3$$

[1]

The coefficient of x will apply a stretch to $y = \cos(x)$, scale factor $\frac{1}{b}$, parallel to the x-axis

To find b, compare the period of 1 wave of the given graph, to the period of $y = \cos(x)$

$$\frac{2\pi}{b} = \frac{8\pi}{3}$$

[1]

$$b = 0.75$$

[1]

This should mean that the point (0, 1) is now at (0, 3) but it is not so the graph has been translated 2 units down

The constant will translate the graph of $y = \cos(x)$ through the vector $\begin{pmatrix} 0 \\ c \end{pmatrix}$

$$c = -2$$

$a = 3$, $b = 0.75$, $c = -2$ [1]

10

Adjust the range $-\frac{\pi}{4} \le B \le \frac{\pi}{4}$ by multiplying the end points by 4 and subtracting $\frac{\pi}{8}$. This will help us decide if our answers will be in the range needed.

$$-3.53429 \le 4B - \frac{\pi}{8} \le 2.74889$$

Rearrange the equation in the question.

$$\sin\left(4B - \frac{\pi}{8}\right) = -\frac{2}{5}$$

Inverse sine both sides.

$$4B - \frac{\pi}{8} = \sin^{-1}\left(-\frac{2}{5}\right)$$

[1]

Work out $\sin^{-1}\left(-\frac{2}{5}\right)$

$$\sin^{-1}\left(-\frac{2}{5}\right) = -0.4115168$$

[1]

This is within our adjusted range and so we can continue with this value.

Therefore:

$$4B - \frac{\pi}{8} = -0.4115168$$

Rearrange and solve.

$$B = -0.0047044$$

To find the other solution in the range, examine the Sine curve or use the CAST method.

$$-\pi + 0.4115168 = -2.73008$$

This is within our adjusted range and so we can continue with this value.

$$4B - \frac{\pi}{8} = -2.73008$$

Rearrange and solve.

$$B = -0.584344$$

$B = -0.00470$ [1]

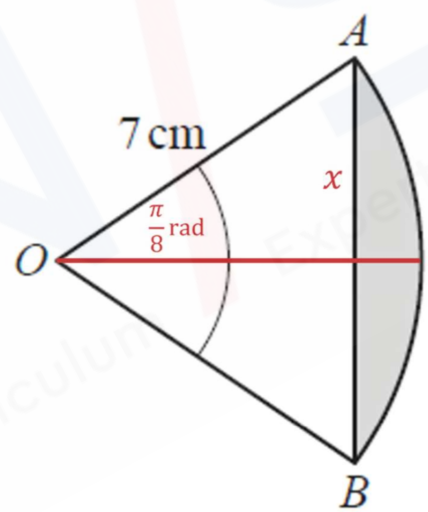
$B = -0.584$ [1]

11

Use the formula $\text{arc length} = r\theta$ to find the length of the arc

$$\text{arc length} = 7 \times \frac{\pi}{4}$$

$$\text{arc length} = \frac{7\pi}{4}$$



Use trigonometry to find the length of the chord

Bisect *AB* and draw a right-angled triangle

Use the sin ratio to find length x

$$\sin\left(\frac{\pi}{8}\right) = \frac{x}{7}$$

$$x = 7\sin\left(\frac{\pi}{8}\right)$$

The length of the chord is double this so multiply by 2

$$\text{chord length} = 14\sin\left(\frac{\pi}{8}\right)$$

The perimeter of the shaded region is the arc length added to the chord length

$$\text{perimeter of shaded region} = 14\sin\left(\frac{\pi}{8}\right) + \frac{7\pi}{4}$$

[1]

10.9 cm [1]

12

i) Find a common denominator which will be $(\sin\theta - 1)(\sin\theta + 1)$

To write the two fractions over this common denominator, we will need to multiply the numerator and denominator of the first fraction by $(sin\theta + 1)$ and the numerator and the denominator of the second fraction by $(\sin\theta - 1)$ giving

$$\frac{\sin\theta + 1}{(\sin\theta - 1)(\sin\theta + 1)} - \frac{\sin\theta - 1}{(\sin\theta + 1)(\sin\theta - 1)}$$

Combine the two separate fractions into one fraction now, since they have the same denominator

$$= \frac{\sin\theta + 1 - (\sin\theta - 1)}{(\sin\theta - 1)(\sin\theta + 1)}$$

[1]

Simplify the numerator by expanding the bracket

$$= \frac{\sin\theta + 1 - \sin\theta + 1}{(\sin\theta - 1)(\sin\theta + 1)}$$

Now we can cancel the $\sin\theta$ terms

$$= \frac{2}{(\sin\theta - 1)(\sin\theta + 1)}$$

Now we can notice that the denominator is the difference of two squares. Use that $(a - b)(a + b) = a^2 - b^2$

$$= \frac{2}{\sin^2\theta - 1}$$

Since $\sin^2\theta + \cos^2\theta = 1$, $\sin^2\theta - 1 = -\cos^2\theta$

$$= \frac{2}{-\cos^2\theta}$$

[1]

$$= -2\sec^2\theta$$

$a = -2$ [1]

ii) Use part (a) with $\theta = 3\phi$

$$\frac{1}{\sin 3\phi - 1} - \frac{1}{\sin 3\phi + 1} = -2\sec^2 3\phi$$

The right-hand side is equal to -8 (from the question) so

$$-2\sec^2 3\phi = -8$$

Solve

$$\sec^2 3\phi = 4$$

Square root

$$\sec 3\phi = \pm 2$$

[1]

Recall that $\sec x = \frac{1}{\cos x}$

$$\frac{1}{\cos 3\phi} = \pm 2$$

$$\cos 3\phi = \pm\frac{1}{2}$$

[1]

Use cos inverse on your calculator in radian mode and use a CAST diagram to find all the angles required. Remember that we will still need to divide by 3 and they should remain in the given range.

$$3\phi = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$$

[1]

Divide by 3

$$\phi = -\frac{2\pi}{9}, -\frac{\pi}{9}, \frac{\pi}{9}, \frac{2\pi}{9}$$

$-\frac{2\pi}{9}, -\frac{\pi}{9}, \frac{\pi}{9}, \frac{2\pi}{9}$ [2]

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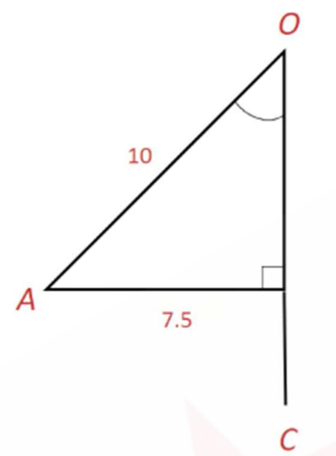
13

Find angle AOC using trigonometry.



$$\text{Angle } AOC = \sin^{-1}\left(\frac{7.5}{10}\right)$$

[1]

$$\text{Angle } AOC = 0.84806$$

Angle AOB is double angle AOC.

$$\text{Angle } AOB = 2 \times 0.848$$

$$\text{Angle } AOB = 1.696$$

AOB $= 1.70$ to 2dp [1]

13b

Use the Cosine rule to work out the length AC.

$$AC^2 = 10^2 + 25^2 - (2 \times 10 \times 25 \times \cos(AOC))$$
$$= 725 - (500 \times \cos(0.848))$$

[1]

$$AC^2 = 394.281$$

$$AC = 19.857$$

[1]

Work out the length of major arc AB.

$$\pi d \times \frac{\text{angle of sector}}{2\pi}$$

$$20\pi \times \frac{(2\pi - 1.7)}{2\pi} = 45.832$$

[1]

Add together the length of major arc AB, and $2 \times$ length AC (since $AC = BC$).

$$45.832 + (2 \times 19.857) = 85.546$$

= 85.55 to 2dp [1]

answers which round to 85.5 or 85.6 are accepted

13c

Find the area of major sector AOB.

$$\pi r^2 \times \frac{\text{angle of sector}}{2\pi}$$

$$\pi \times (10)^2 \times \frac{(2\pi - 1.7)}{2\pi}$$

[1]

$$= 229.159$$

[1]

Find the area of kite AOBC.

$$\frac{15 \times 25}{2} = 187.5$$

[1]

Add together the area of the major sector and the area of the kite.

$$229.159 + 187.5$$

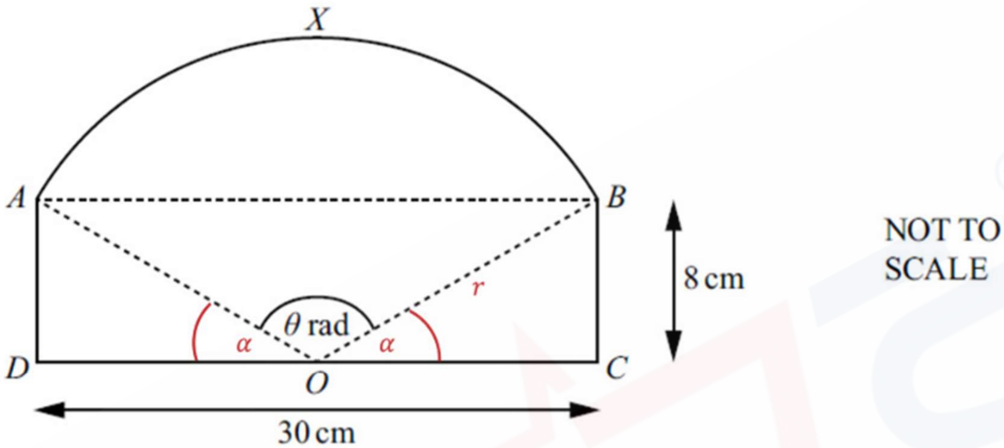
[1]

416.659 [1]

answers which round to 417 are accepted

14a

Let's label the radius of the sector as r and angles BOC and DOC as α. $DO = OC = \frac{30}{2} = 15$ cm



We can find r using Pythagoras theorem

$$r = \sqrt{8^2 + 15^2}$$

[1]

$$= 17$$

We can find α using right-angle trigonometry. θ is in radians so remember to set your calculator to radians

$$\alpha = \tan^{-1}\left(\frac{8}{15}\right) = 0.489957326...$$

Now find θ by subtracting 2α from π

$$\theta = \pi - 2(0.489957326...)$$

[1]

$$= 2.16167800...$$

[1]

(It's worth saving this result in your calculator as it will be used more than once in this question)

Now find the length of the arc using arc length = θr (or $\frac{\theta}{2\pi}(2\pi r)$)

arc length $= 2.16167800... \times 17$

And finally find the perimeter of $ADOCBX$ by adding the 3 straight lengths to the arc length

$$\text{perimeter} = 8 + 8 + 30 + 2.16167800... \times 17$$

[1]

$$= 82.748560...$$

82.7 cm (3 s.f.) [1]

14b

We know from part (a) that $\theta = 2.16167800...$

Find the area of the sector *OAXB*

$$\frac{1}{2} \times 17^2 \times 2.16167800... = 312.362...$$

Find the area of the triangle *OBC* (and *OAD*)

$$\frac{1}{2} \times 15 \times 8 = 60$$

Add together the area of the sector and the area of the two triangles

$$\text{Total area} = \frac{1}{2} \times 17^2 \times 2.16167800... + 60 + 60$$
$$= 432.362471...$$

[1]

Round the final answer to at least 3 significant figures

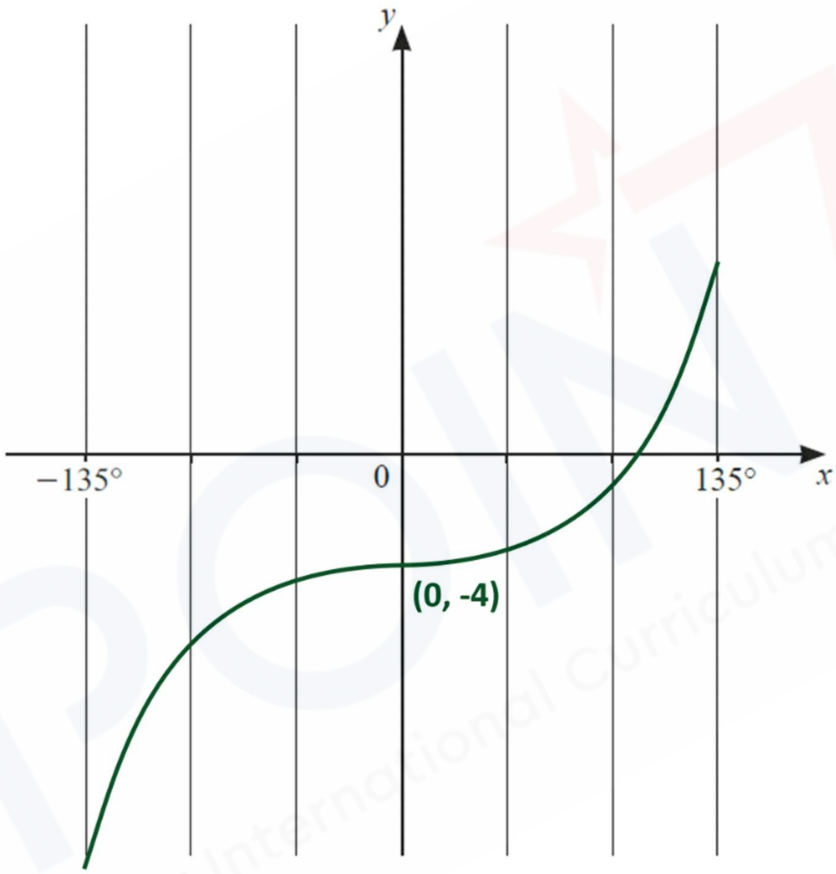
432 cm^2 (3 s.f.) [1]

15

$\frac{x}{2}$ is a horizontal stretch of scale factor 2, therefore the asymptotes of $\pm 90°$ are now at $\pm 180°$ which is out of the given range. Because of this, we will only be drawing one continuous section.

To get the y intercept, substitute $x = 0$ into the equation.

$$y = 3\tan\left(\frac{0}{2}\right) - 4$$
$$y = -4$$



for correct tan shape [1]

for correct y intercept $(0, -4)$ [1]

16a

Since arc length, in radians, is $r\theta$ and we are told that the arc length *BC* is $1.5r$

$$\text{Angle } BOC = 1.5 \text{ radians}$$

[1]

Use trigonometry to find an expression for the length *BC* in terms of r.

$$\sin(0.75) = \frac{BC/2}{r}$$

[1]

Rearrange to make BC the subject.

$$BC = 2r \times \sin(0.75)$$

[1]

Add together the lengths around the perimeter of the shaded area.

$$2r + 4r + 1.5r + (2r\sin(0.75))$$

[1]

Simplify.

$$7.5r + 2r\sin(0.75)$$

Factorise by pulling out a factor of r.

$(7.5 + 2\sin 0.75)r$ [1]

16b

Find the area of the rectangle by adding together the lengths *AB*, *BC* and *CD*, and multiplying it by *DE*.

$$\text{Area (rectangle)} = (r + 2r\sin 0.75 + r) \times r$$

Simplify.

$$=(2r+2r\sin 0.75)r$$

[1]

Write in terms of r^2

$$=2r^2+2r^2(\sin 0.75)$$
$$=r^2(2+2\sin 0.75)$$
$$=3.36r^2$$

Find the area of the segment by subtracting the area of triangle *BOC* from the area of the sector.

$$\text{Area (sector)}=\frac{1}{2}r^2\theta$$
$$=\frac{1}{2}r^2(1.5)$$

$$\text{Area (triangle)}=\frac{1}{2}ab\sin C$$
$$=\frac{1}{2}r^2(\sin 1.5)$$

$$\text{Area (segment)}=\frac{1}{2}r^2(1.5)-\frac{1}{2}r^2(\sin 1.5)$$

Factorise by pulling out a factor of $\frac{1}{2}r^2$ and simplify to write in terms of r^2

$$\text{Area (segment)}=\frac{1}{2}r^2(1.5-\sin 1.5)$$
$$=0.251r^2$$

[1]

Subtract the area of the segment from the area of the rectangle.

$$\text{Area (shaded)}=3.36r^2-0.251r^2$$

[1]

$3.11r^2$ [1]

17a

The total perimeter is 4π so the perimeter of one arc AB is 2π

Find the angle AC_1C_2

AC_1 is a radius and AC_2 is also a radius, and C_1C_2 is a radius too meaning that triangle AC_1C_2 is an equilateral triangle so

$$\text{angle } AC_1C_2 = 60° = \frac{\pi}{3}$$

Use this to find angle ACB

$$\text{angle } ACB = 2 \times \frac{\pi}{3}$$

$$\text{angle } ACB = \frac{2\pi}{3}$$

[1]

Apply the formula for arc length using half of the perimeter

$$\text{arc length} = r\theta$$

$$2\pi = r \times \frac{2\pi}{3}$$

correct arc length 2π [1]

correct equation [1]

Multiply by 3

$$2\pi r = 6\pi$$

Divide by 2π

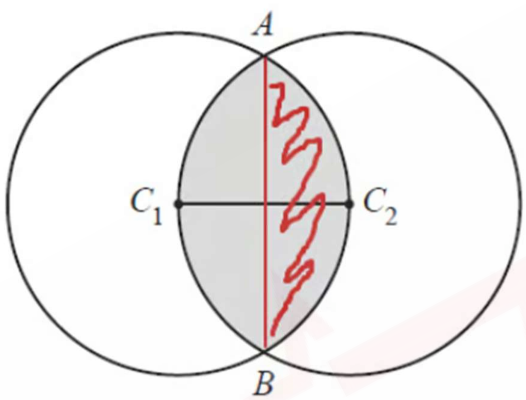
$$r = 3$$

$r = 3$ [1]

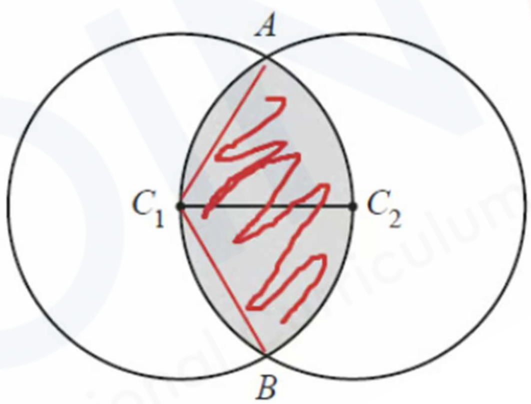
17b

From part (a) $r = 3$ and angle $ACB = \frac{2\pi}{3}$

Split the area in half by drawing a line connecting A and B



Find the red shaded area by first working out the area of the sector below



Use the area of a sector formula, area of sector $= \frac{1}{2}r^2\theta$

$$\text{area of sector } AC_1B = \frac{1}{2} \times 3^2 \times \frac{2\pi}{3}$$

[1]

$$\text{area of sector } AC_1B = 3\pi$$

Now find the area of the triangle using area $= \frac{1}{2}ab\sin C$

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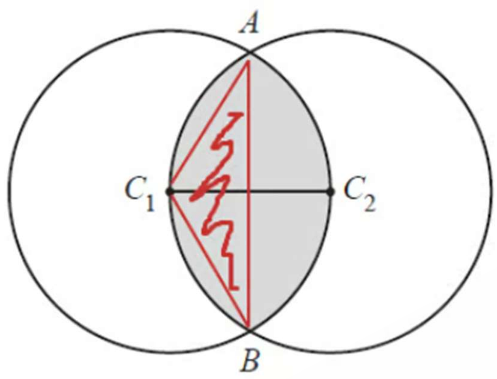


$$\text{area of triangle} = \frac{1}{2} \times 3 \times 3 \times \sin\left(\frac{2\pi}{3}\right)$$

[1]

$$\text{area of triangle} = \frac{9\sqrt{3}}{4}$$

Subtract the triangle from the sector to get the shaded area in the first diagram

$$3\pi - \frac{9\sqrt{3}}{4}$$

[1]

We need 2 of these so double your answer to get the total shaded area

$$2\left(3\pi - \frac{9\sqrt{3}}{4}\right)$$

$$6\pi - \frac{9\sqrt{3}}{2}$$ [1]

18

Rewrite the equation as an equation in tan using $\cot x = \frac{1}{\tan x}$

$$\frac{1}{\tan\left(y - \frac{\pi}{2}\right)} = \sqrt{3}$$

$$\therefore \tan\left(y - \frac{\pi}{2}\right) = \frac{1}{\sqrt{3}}$$

[1]

Use the inverse tan button on your calculator to find $y - \frac{\pi}{2}$ - make sure your calculator is in radians!

$$y - \frac{\pi}{2} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{\pi}{6}$$

[1]

Check for other solutions. Adjust the domain

$$0 \le y \le \pi$$

$$-\frac{\pi}{2} \le y - \frac{\pi}{2} \le \frac{\pi}{2}$$

$$\frac{\pi}{6} + \pi > \frac{\pi}{2} \text{ and } \frac{\pi}{6} - \pi < -\frac{\pi}{2} \therefore \text{ no other solutions within the domain}$$

So now we just need to solve $y - \frac{\pi}{2} = \frac{\pi}{6}$

$$y = \frac{\pi}{6} + \frac{\pi}{2}$$

$\frac{2\pi}{3}$ [1]

"2.09" or "2.09439510..." rounded correctly to at least 3 significant figures is accepted

19

Start with the left hand side and use $\tan x = \dfrac{\sin x}{\cos x}$

$$\frac{\sin x \tan x}{1 - \cos x} = \frac{\sin x \times \dfrac{\sin x}{\cos x}}{1 - \cos x}$$

[1]

Multiply the numerator and denominator by $\cos x$

$$= \frac{\sin^2 x}{\cos x(1 - \cos x)}$$

Use $\sin^2 x + \cos^2 x \equiv 1$ in the numerator

$$= \frac{1 - \cos^2 x}{\cos x(1 - \cos x)}$$

[1]

Factorise the numerator as it is the difference of 2 squares

$$= \frac{(1 - \cos x)(1 + \cos x)}{\cos x(1 - \cos x)}$$

[1]

$$= \frac{1+\cos x}{\cos x}$$

Separate the fraction into two fractions

$$= \frac{1}{\cos x} + \frac{\cos x}{\cos x}$$

$$= \sec x + 1$$

$$\therefore \frac{\sin x \tan x}{1 - \cos x} = 1 + \sec x$$

[1]

20

i) We can rewrite the numerator using the Pythagorean trig identity $\cos^2 A = 1 - \sin^2 A$ ($\sin^2 A + \cos^2 A = 1$ is given on the formula sheet)

$$\frac{\cos^2 2x}{1+\sin 2x} = \frac{1-\sin^2 2x}{1+\sin 2x}$$

[1]

We can rewrite the numerator using the difference of two squares and cancel the fraction

$$\frac{(1+\sin 2x)(1-\sin 2x)}{1+\sin 2x} = 1 - \sin 2x$$ [1]

ii) The left hand side of the equation is $3\left(\frac{\cos^2 2x}{1+\sin 2x}\right)$. Therefore

$$\frac{3\cos^2 2x}{1+\sin 2x} = 3(1-\sin 2x) = 1$$

Now solve

$$1 - \sin 2x = \frac{1}{3}$$

$$\sin 2x = \frac{2}{3}$$

[1]

The domain is given in degrees so make sure your calculator is in degrees

$$2x = \sin^{-1}\left(\frac{2}{3}\right) = 41.81031...^\circ$$

Divide both sides by two to get the first solution

$$x_1 = 20.905157...$$

[1]

The domain is for x so double it to get the domain for $2x$

$$0^\circ \le 2x \le 180^\circ$$

Use the trig identity $\sin A = \sin(180° - A)$ to find the second solution

$$2x_2 = 180^\circ - 41.81031...^\circ = 138.189685...^\circ$$

Divide by 2

$$x_2 = 69.09484...$$

20.9°, 69.1° (3 s.f.) [2]

one mark for each correct answer correctly rounded to at least 3 significant figures

21

Substitute $\frac{1}{\cos\left(2\phi + \frac{\pi}{4}\right)}$ for $\sec\left(2\phi + \frac{\pi}{4}\right)$

$$\frac{1}{2} \times \frac{1}{\cos\left(2\phi + \frac{\pi}{4}\right)} = \frac{1}{\sqrt{3}}$$

Multiply the fractions on the left hand side.

$$\frac{1}{2\cos\left(2\phi + \frac{\pi}{4}\right)} = \frac{1}{\sqrt{3}}$$

Multiply both sides by 2.

$$\frac{1}{\cos\left(2\phi + \frac{\pi}{4}\right)} = \frac{2}{\sqrt{3}}$$

Take the reciprocal of both sides.

$$\cos\left(2\phi + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$$

[1]

Take $\cos^{-1}$ of both sides.

$$2\phi + \frac{\pi}{4} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$2\phi + \frac{\pi}{4} = \frac{\pi}{6}$$

Rearrange and solve to find one solution.

$$2\phi = \frac{\pi}{6} - \frac{\pi}{4}$$

$$2\phi = -\frac{\pi}{12}$$

$$\phi = -\frac{\pi}{24}$$

[1]

The range given in the question is $-\pi < \phi < \pi$. If we rewrite this for $2\phi + \frac{\pi}{4}$, our range will be

$$-\frac{7\pi}{4} < 2\phi + \frac{\pi}{4} < \frac{9\phi}{4}$$

By either using CAST method, or by examining the Cosine curve, find the other solutions to $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$, ignoring the ones outside of our range.

$$\frac{13\pi}{6}, \quad \frac{11\pi}{6}, \quad -\frac{\pi}{6}$$

Substitute each of these values into $2\phi + \frac{\pi}{4} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$, rearrange and solve.

$$2\phi + \frac{\pi}{4} = \frac{13\pi}{6}$$

$$2\phi = \frac{13\pi}{6} - \frac{\pi}{4}$$

$$\phi = \frac{23\pi}{24}$$

[1]

$$2\phi = \frac{11\pi}{6} - \frac{\pi}{4}$$

$$\phi = \frac{19\pi}{24}$$

[1]

$$2\phi + \frac{\pi}{4} = -\frac{\pi}{6}$$

$$2\phi = -\frac{\pi}{6} - \frac{\pi}{4}$$

$$\phi = -\frac{5\pi}{24}$$

$\phi = -\frac{5\pi}{24}$, $\phi = -\frac{\pi}{24}$, $\phi = \frac{19\pi}{24}$, $\phi = \frac{23\pi}{24}$ [1]

22a

For any function in the form $a\cos(bx) + c$, the amplitude is given by a

2 [1]

22b

For any function in the form $a\cos(bx) + c$, the period is $\frac{2\pi}{b}$ in radians or $\frac{360°}{b}$ in degrees. As the units (degrees or radians) is not specified in the question you can use either

Radians:

$$\text{period} = \frac{2\pi}{\frac{1}{3}} = 2\pi \times 3$$

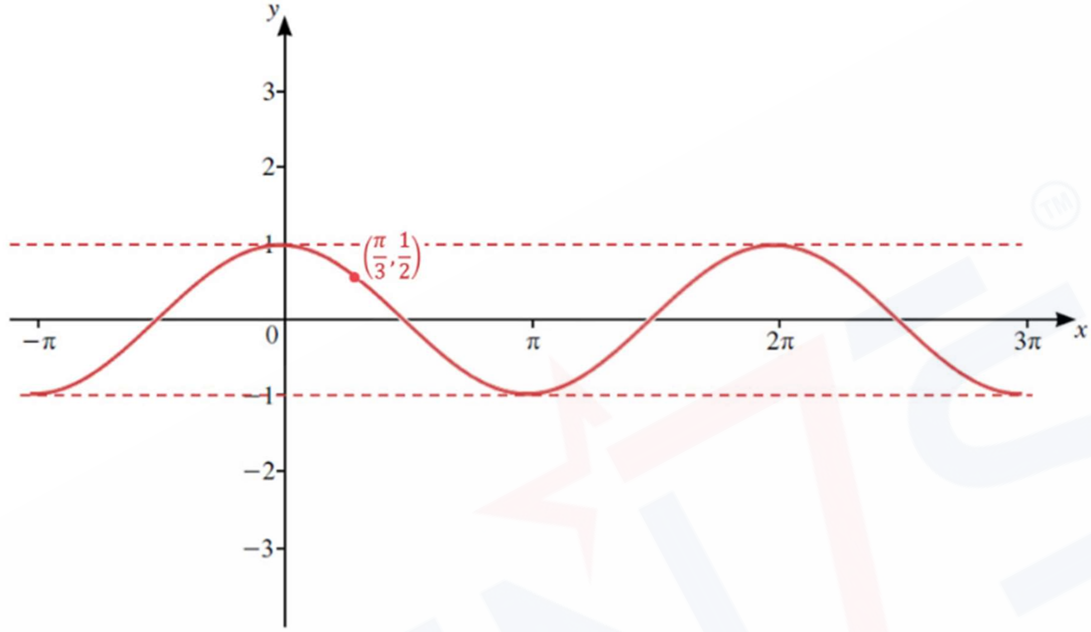
Degrees:

$$\text{period} = \frac{360°}{\frac{1}{3}} = 360° \times 3$$

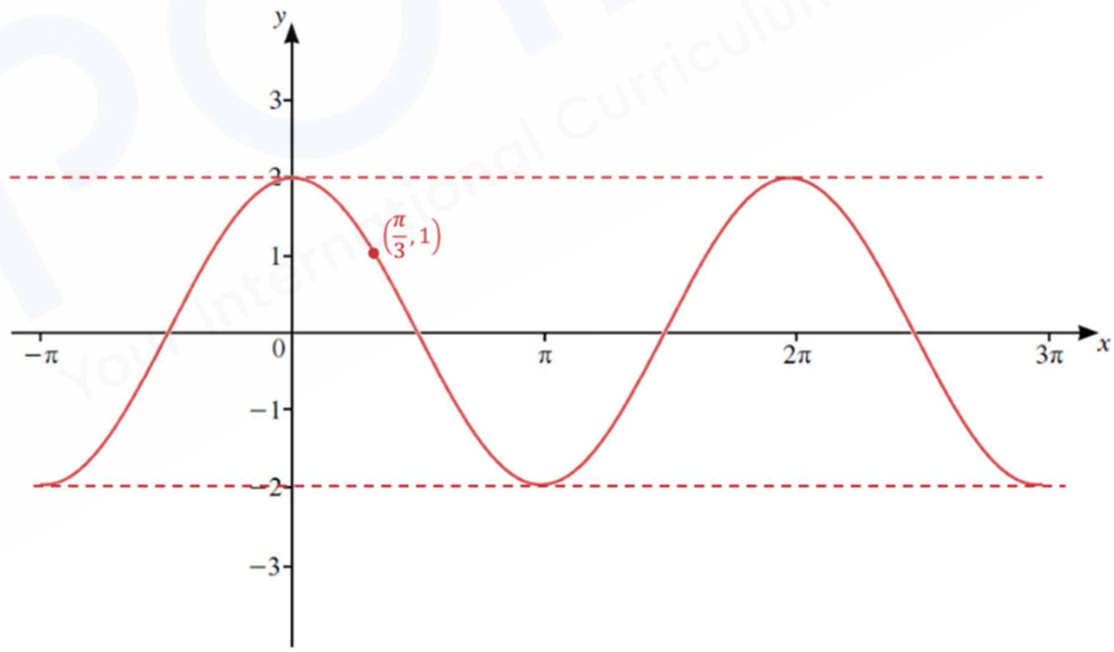
6π or 1080° [1]

22c

Start by sketching the graph of $y = \cos x$. Note that when $x = \frac{\pi}{3}$, $y = \frac{1}{2}$



We know from part (a) that the amplitude is 2, so stretch the graph in the y-axis by 2. Note that when $x = \frac{\pi}{3}$, $y = 1$



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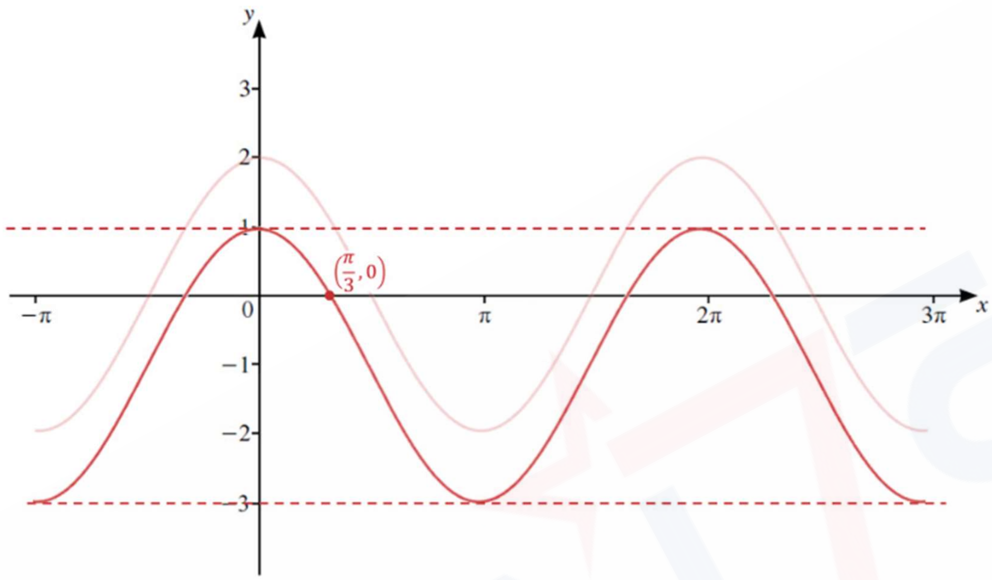
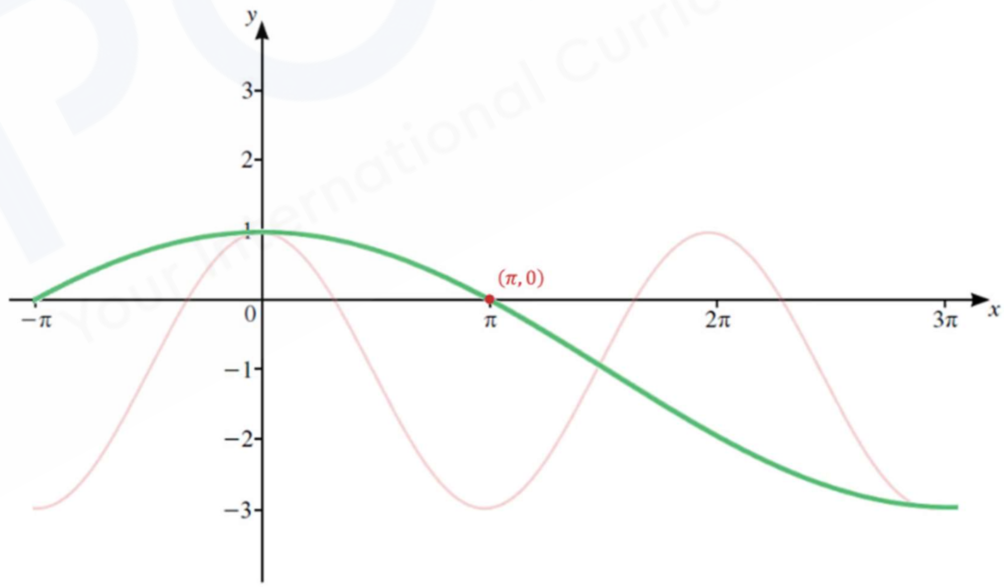
The "-1" in $2\cos\frac{x}{3}-1$ (or the c in $a\cos(bx)+c$) tells us that the graph of $y=2\cos x$ is translated down 1. Note that now, when $x=\frac{\pi}{3}$, $y=0$



We know from part (b) that the period of the graph is 6π rather than 2π. Therefore the graph is stretched by a factor of 3 in the x-axis. In other words, the y-coordinates stay the same but the x-coordinates are multiplied by 3

So $\left(\frac{\pi}{3},0\right)$ becomes $(\pi,0)$, and $(\pi,-3)$ becomes $(3\pi,-3)$



Note that due to the symmetry of the curve, if we know the curve passes through $(\pi, 0)$ then we should also have it passing through $(-\pi, 0)$

curve passing through $(-\pi, 0)$ and $(3\pi, -3)$ [1]
correct shape with maximum point on *y*-axis and a minimum at $x = 3\pi$ [1]
passes through $(0, 1)$ and $(\pi, 0)$ with no other positive *x*-intercepts [1]