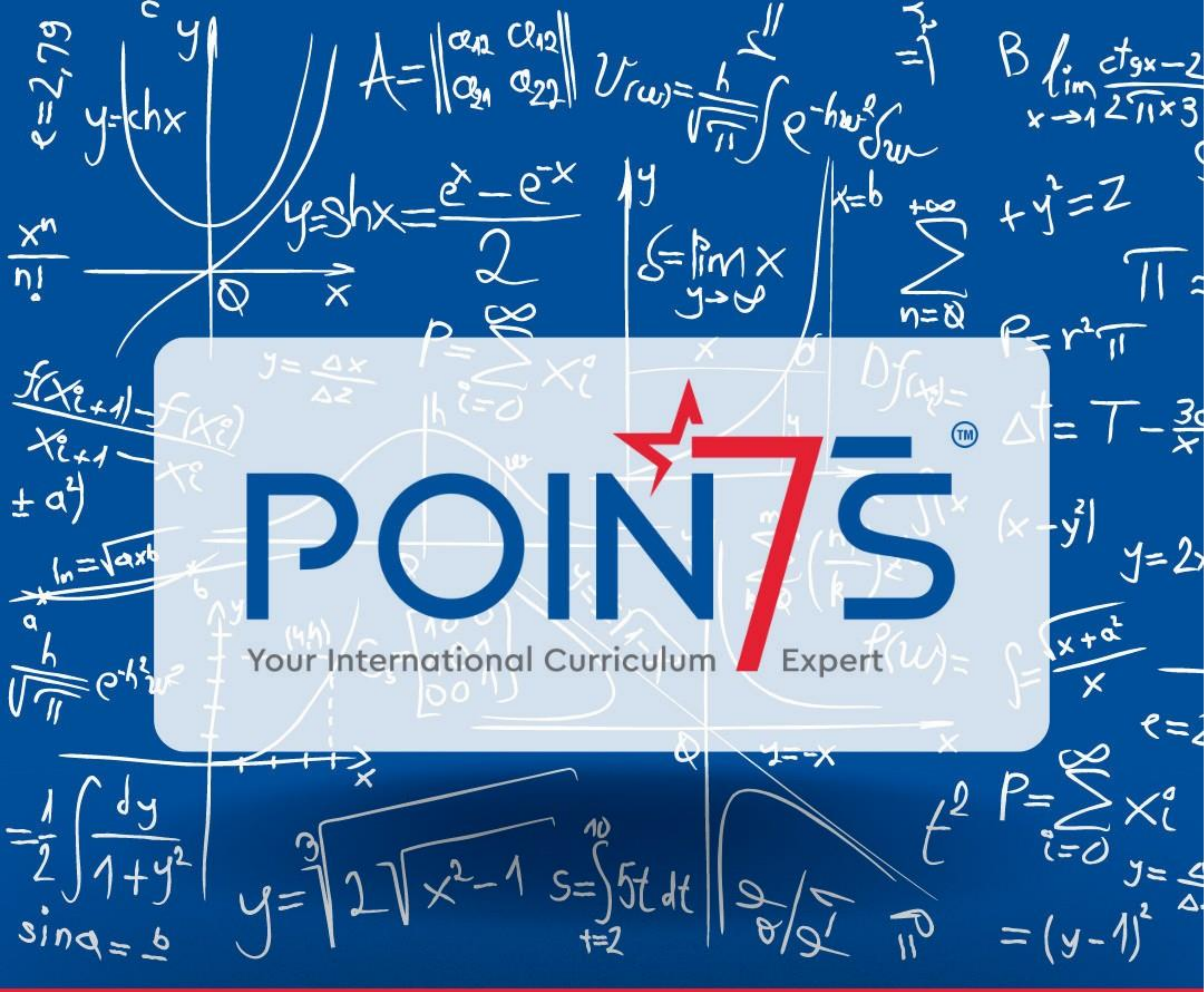
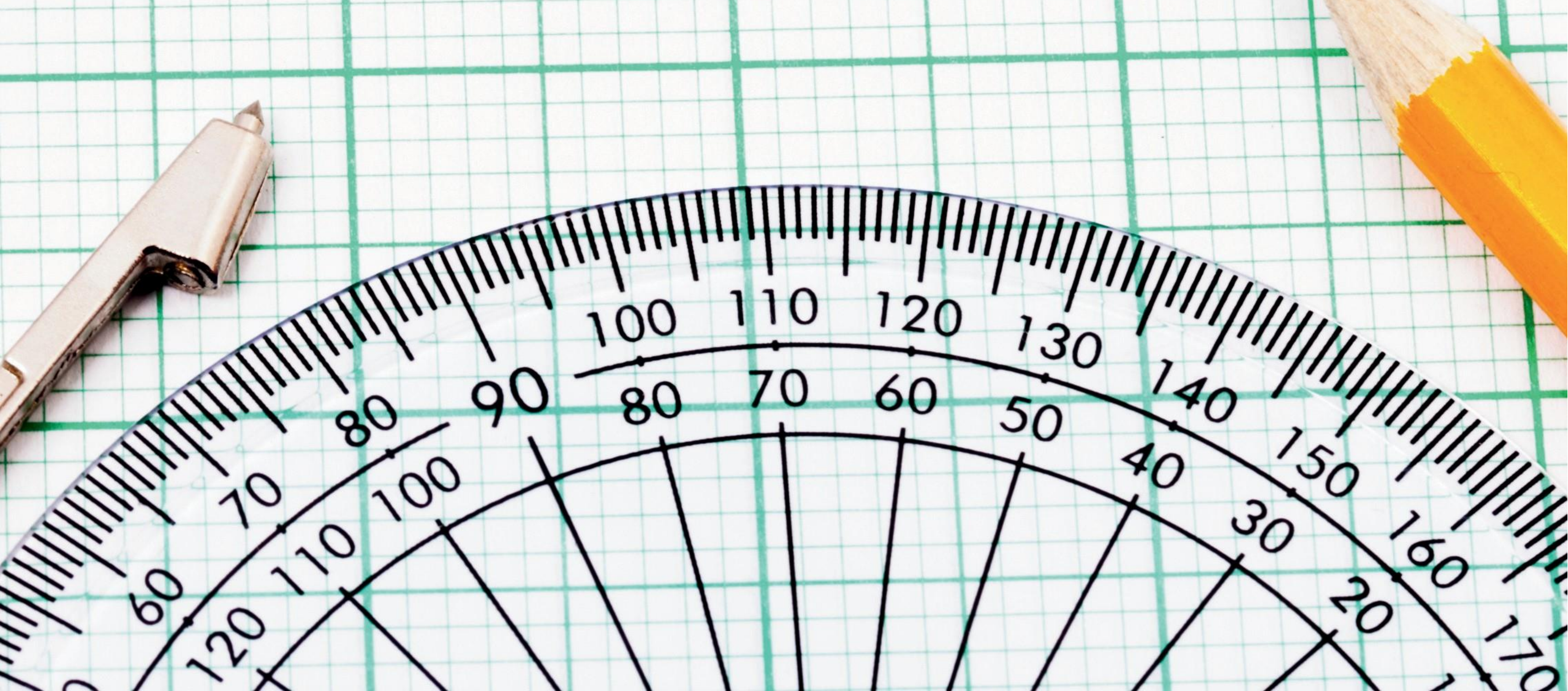


ELEVATE

MATH TOPICAL WORKSHEETS

IGCSE- Additional Mathematics (0606)

Calculus,Kinematics and Vectors



1 (a) Show that $\frac{3}{2x-3}+\frac{3}{2x+3}$ can be written as $\frac{12x}{4x^2-9}$.

(2 marks)

(b) Hence find $\int \frac{12x}{4x^2-9}\,dx$, giving your answer as a single logarithm and an arbitrary constant.

(3 marks)

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(c) Given that $\int_2^a \frac{12x}{4x^2 - 9} \mathrm{d}x = \ln 5\sqrt{5}$, where $a > 2$, find the exact value of a.

(4 marks)

2 (a) A curve has equation $y = \dfrac{\ln(3x^2 - 5)}{2x + 1}$ for $3x^2 > 5$

Find the equation of the normal to the curve at the point where $x = \sqrt{2}$.

(6 marks)

(b) Find the approximate change in y as x increases from $\sqrt{2}$ to $\sqrt{2} + h$, where h is small.

(1 mark)

3 (a) Find the equation of the tangent to the curve $y = x^3 - 6x^2 + 3x + 10$ at the point where $x = 1$.

(4 marks)

(b) Find the coordinates of the point where this tangent meets the curve again.

(5 marks)

4 (a) It is given that $y = \dfrac{\tan 3x}{\sin x}$.

Find the exact value of $\dfrac{dy}{dx}$ when $x = \dfrac{\pi}{3}$.

(4 marks)

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(b) Hence find the approximate change in y as x increases from $\frac{\pi}{3}$ to $\frac{\pi}{3} + h$, where h is small.

(1 mark)

(c) Given that x is increasing at the rate of 3 units per second, find the corresponding rate of change in y when $x = \frac{\pi}{3}$, giving your answer in its simplest surd form.

(2 marks)

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5 (a) A particle moves in a straight line such that, t seconds after passing a fixed point O, its displacement from O is s m, where $s = e^{2t} - 10e^{t} - 12t + 9$.

Find expressions for the velocity and acceleration at time t.

(3 marks)

(b) Find the time when the particle is instantaneously at rest.

(3 marks)

(c) Find the acceleration at this time.

(2 marks)

6 (a) A particle travels in a straight line. As it passes through a fixed point O, the particle is travelling at a velocity of $3\ \text{ms}^{-1}$. The particle continues at this velocity for 60 seconds then decelerates at a constant rate for 15 seconds to a velocity of $1.6\ \text{ms}^{-1}$. The particle then decelerates again at a constant rate for 5 seconds to reach point A , where it stops.

Sketch the velocity-time graph for this journey on the axes below.

(3 marks)

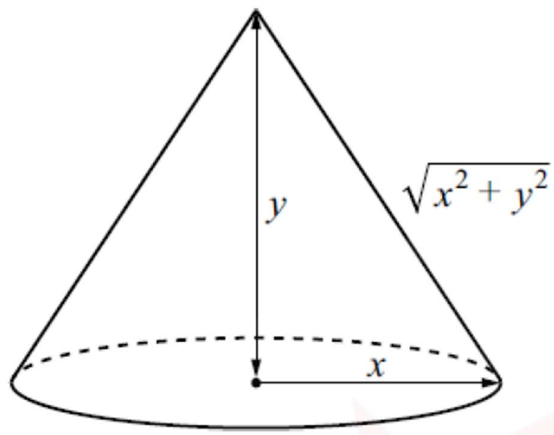
(b) Find the distance between O and A .

(3 marks)

(c) Find the deceleration in the last 5 seconds.

(1 mark)

7 (a) In this question, all lengths are in centimetres.



The diagram shows a cone of base radius x, height y and sloping edge $\sqrt{x^2+y^2}$. The volume of the cone is 10π cm^3.

Show that the curved surface area, S, of the cone is given by $S = \dfrac{\pi\sqrt{x^6+900}}{x}$.

(3 marks)

(b) Given that x can vary and that S has a minimum value, find the value of x for which S is a minimum.

(5 marks)

8 (a) A curve has equation $y = x \cos x$.

Find $\frac{dy}{dx}$.

(2 marks)

(b) Find the equation of the normal to the curve at the point where $x = \pi$, giving your answer in the form $y = mx + c$.

(4 marks)

9 (a) Given that $y = (x^2 - 1)\sqrt{5x + 2}$, show that $\frac{dy}{dx} = \frac{Ax^2 + Bx + C}{2\sqrt{5x + 2}}$, where A, B and C are integers.

(5 marks)

(b) Find the coordinates of the stationary point of the curve $y = (x^2 - 1)\sqrt{5x + 2}$ for $x > 0$.
Give each coordinate correct to 2 significant figures.

(3 marks)

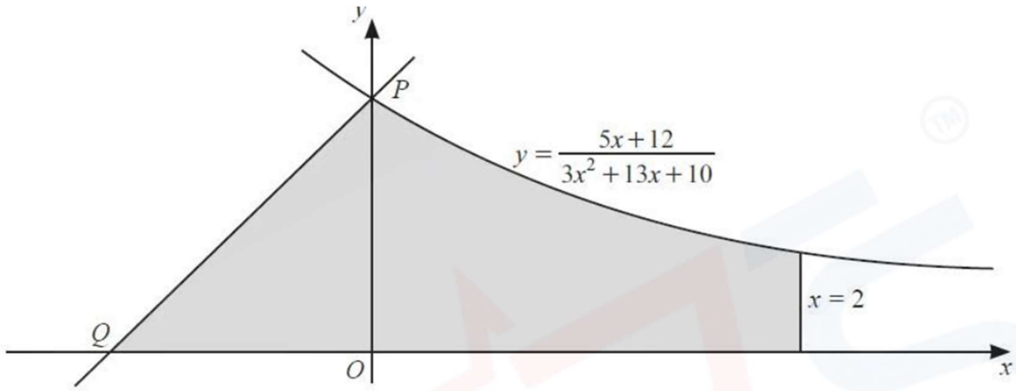
(c) Determine the nature of this stationary point.

(2 marks)

10 (a) Show that $\dfrac{1}{x+1} + \dfrac{2}{3x+10}$ can be written as $\dfrac{5x+12}{3x^2+13x+10}$

(1 mark)

(b)



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The diagram shows part of the curve $y=\frac{5x+12}{3x^2+13x+10}$, the line $x=2$ and a straight line of gradient 1. The curve intersects the y-axis at the point P. The line of gradient 1 passes through P and intersects the x-axis at the point Q. Find the area of the shaded region, giving your answer in the form $a+\frac{2}{3}\ln\left(b\sqrt{3}\right)$, where a and b are constants.

(9 marks)

11 (a) Giving your answer in its simplest form, find the exact value of

$$\int_0^4 \frac{10}{5x+2}\,dx$$

(4 marks)

(b) $\int_0^{\ln 2} (e^{4x}+2)^2 \, dx$

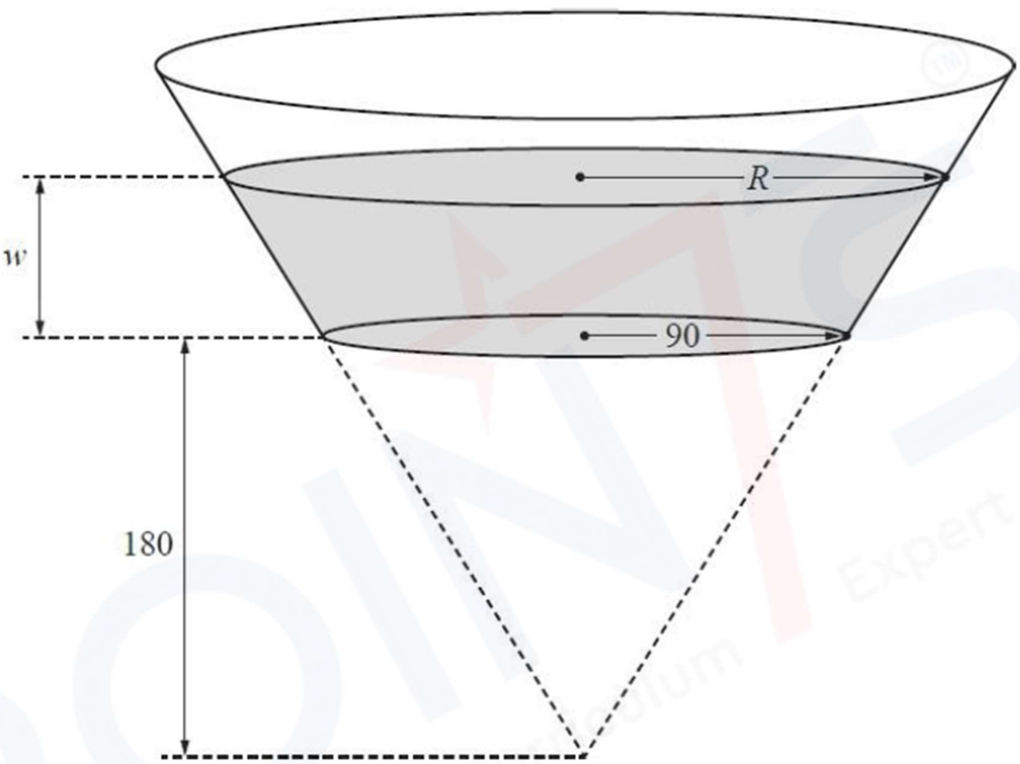
(5 marks)

12 Find the equation of the tangent to the curve $y = \dfrac{\ln(3x^2 - 1)}{x + 2}$ at the point where $x = 1$.

Give your answer in the form $y = mx + c$, where m and c are constants correct to 3 decimal places.

(6 marks)

13 (a) In this question all lengths are in centimetres. The volume, V, of a cone of height h and base radius r is given by $V = \frac{1}{3}\pi r^2 h$



The diagram shows a large hollow cone from which a smaller cone of height 180 and base radius 90 has been removed. The remainder has been fitted with a circular base of radius 90 to form a container for water. The depth of water in the container is w and the surface of the water is a circle of radius R.

Find an expression for R in terms of w and show that the volume V of the water in the container is given by $V = \frac{\pi}{12}(w + 180)^3 - 486000\pi$.

(3 marks)

(b) Water is poured into the container at a rate of $10\ 000\ \mathrm{cm^3 s^{-1}}$. Find the rate at which the depth of the water is increasing when $w = 10$.

(4 marks)

14 (a) A curve has the equation $y = e^{(4x - x^2)}$.

Find and factorise an expression for $\frac{dy}{dx}$.

(2 marks)

(b) Hence find $\int \frac{(2-x)e^{4x}}{e^{x^2}} dx$.

(2 marks)

(c) Show that the second derivative of the curve $y = e^{(4x - x^2)}$ satisfies the relationship

$$\frac{d^2y}{dx^2} = (p + qx + rx^2)y$$

where p , q and r are constants to be found.

(4 marks)

15 A curve is such that $\frac{d^2y}{dx^2} = \sin\left(6x - \frac{\pi}{2}\right)$. Given that $\frac{dy}{dx} = \frac{1}{2}$ at the point $\left(\frac{\pi}{4}, \frac{13\pi}{12}\right)$ on the curve, find the equation of the curve.

(7 marks)

16 (a) Differentiate $y = \tan(x+4) - 3\sin x$ with respect to x.

(2 marks)

(b) Variables x and y are such that $y = \dfrac{\ln(2x+5)}{2e^{3x}}$. Use differentiation to find the approximate change in y as x increases from 1 to $1 + h$, where h is small.

(6 marks)

17

(a) The vectors $\mathbf{a}$ and $\mathbf{b}$ are such that $\mathbf{a} = \alpha\mathbf{i} + \mathbf{j}$ and $\mathbf{b} = 12\mathbf{i} + \beta\mathbf{j}$.

Find the value of each of the constants α and β such that $4\mathbf{a} - \mathbf{b} = (\alpha + 3)\mathbf{i} - 2\mathbf{j}$.

(3 marks)

(b) Hence find the unit vector in the direction of $\mathbf{b} - 4\mathbf{a}$.

(2 marks)

18

(a) The unit vectors $\mathbf{i}$ and $\mathbf{j}$ represent due east and due north respectively.

Person A starts at a position of $(-4\mathbf{i}+\mathbf{j})$ metres and walks with a constant velocity of $(3\mathbf{i}-\mathbf{j})$ metres per second.

Find the position vector of person A after 5 seconds.

(1 mark)

(b) Person B walks with a constant velocity of $(2\mathbf{i}+2\sqrt{3}\mathbf{j})$ metres per second.

Find

(i) the speed of person B,

(ii) the bearing of the direction in which person B is walking.

(5 marks)

19

The position vectors of three points, A, B and C, relative to an origin O, are $\begin{pmatrix}-5\\-7\end{pmatrix}$, $\begin{pmatrix}10\\-4\end{pmatrix}$ and $\begin{pmatrix}x\\y\end{pmatrix}$ respectively. Given that $\overrightarrow{AC} = 4\overrightarrow{BC}$, find the unit vector in the direction of $\overrightarrow{OC}$.

(5 marks)

20

(a) Find the unit vector in the direction of $\begin{pmatrix} 5 \\ -12 \end{pmatrix}$.

(1 mark)

(b) Given that $\begin{pmatrix} 4 \\ 1 \end{pmatrix} + k\begin{pmatrix} -2 \\ 3 \end{pmatrix} = r\begin{pmatrix} -10 \\ 5 \end{pmatrix}$, find the value of each of the constants k and r.

(3 marks)

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(c) Relative to an origin O, the points A, B and C have position vectors $\mathbf{p}$, $3\mathbf{q}-\mathbf{p}$ and $9\mathbf{q}-5\mathbf{p}$ respectively.

(i) Find $\overrightarrow{AB}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

[1]

(ii) Find $\overrightarrow{AC}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

[1]

(iii) Explain why A, B and C all lie in a straight line.

[1]

(iv) Find the ratio $AB : BC$.

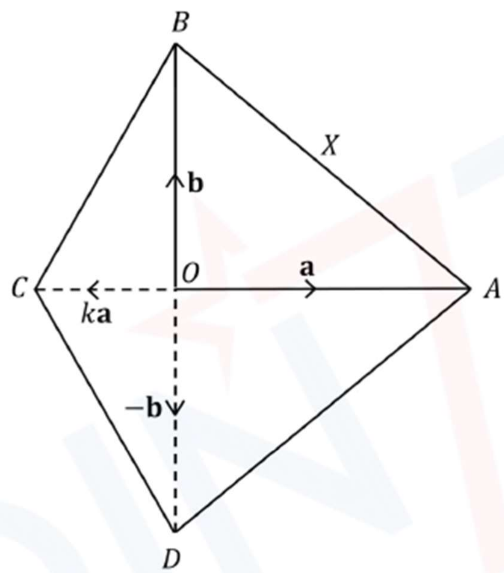
[1]

(4 marks)

21

; **(a)** The kite $ABCD$ has diagonals CA and BD which intersect at O, where $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OC} = k\mathbf{a}$ and $\overrightarrow{OD} = -\mathbf{b}$.

The point X is the midpoint of AB, as shown.



Find

(i) $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

(ii) $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$

(iii) $\overrightarrow{CD}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$

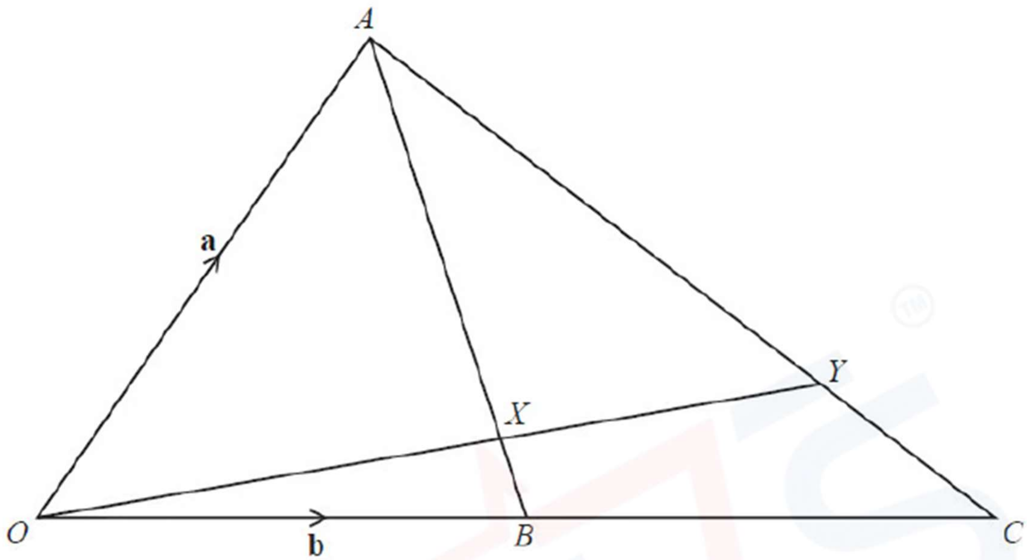
(b) The point Y lies on CD such that $CY : YD = 1:2$.

If $\overrightarrow{YX}$ is parallel to $\mathbf{a} + \mathbf{b}$, find k.

(4 marks)

22

(a)



The diagram shows the triangle OAC. The point B is the midpoint of OC. The point Y lies on AC such that OY intersects AB at the point X where $AX:XB = 3:1$. It is given that $\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$.

Find $\overrightarrow{OX}$ in terms of a and b, giving your answer in its simplest form.

(3 marks)

(b) Find *AC* in terms of **a** and **b**.

(1 mark)

(c) Given that $\overrightarrow{OY} = h\overrightarrow{OX}$, find $\overrightarrow{AY}$ in terms of a, b and h.

(1 mark)

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(d) Given that $\overrightarrow{AY} = m\overrightarrow{AC}$, find the value of h and of m.

(4 marks)

22

(a) In this question all distances are in km.

A ship P sails from a point A, which has position vector $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, with a speed of $52\ \text{kmh}^{-1}$ in the direction of $\begin{pmatrix} -5 \\ 12 \end{pmatrix}$

Find the velocity vector of the ship.

(1 mark)

(b) Write down the position vector of P at a time t hours after leaving A.

(1 mark)

(c) At the same time that ship P sails from A, a ship Q sails from a point B, which has position vector $\begin{pmatrix} 12 \\ 8 \end{pmatrix}$, with velocity vector $\begin{pmatrix} -25 \\ 45 \end{pmatrix} \text{kmh}^{-1}$.

Write down the position vector of Q at a time t hours after leaving B.

(1 mark)

(d) Using your answers to parts (b) and (c), find the displacement vector $\overrightarrow{PQ}$ at time t hours.

(1 mark)

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(e) Hence show that $PQ = \sqrt{34t^2 - 168t + 208}$.

(2 marks)

(f) Find the value of t when P and Q are first 2 km apart.

(2 marks)

23

The parallelogram $OABC$ is such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. The point D lies on OC such that $OD : DC = 1 : 2$. The point E lies on AC such that $AE : EC = 2 : 1$.

Show that $\overrightarrow{OB} = k\overrightarrow{DE}$, where k is an integer to be found.

(5 marks)

24

(a) A particle P is initially at the point with position vector $\begin{pmatrix} 30 \\ 10 \end{pmatrix}$ and moves with a constant speed of 10 ms^{-1} in the same direction as $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$.

Find the position vector of P after t s.

(3 marks)

(b) As P starts moving, a particle Q starts to move such that its position vector after t s is given by $\begin{pmatrix} -80 \\ 90 \end{pmatrix} + t\begin{pmatrix} 5 \\ 12 \end{pmatrix}$.

Write down the speed of Q.

(1 mark)

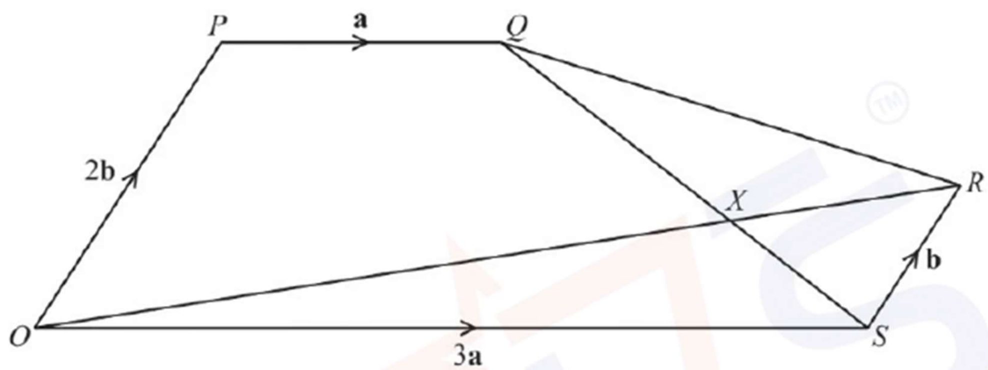
(c) Find the exact distance between P and Q when $t = 10$, giving your answer in its simplest surd form.

(3 marks)

25

(a) In the diagram $\overrightarrow{OP} = 2\mathbf{b}$, $\overrightarrow{OS} = 3\mathbf{a}$, $\overrightarrow{SR} = \mathbf{b}$ and $\overrightarrow{PQ} = \mathbf{a}$. The lines OR and QS intersect at X.

Find $\overrightarrow{OQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.



(1 mark)

(b) Find $\overrightarrow{QS}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(1 mark)

(c) Given that $\overrightarrow{QX} = \mu\overrightarrow{QS}$, find $\overrightarrow{OX}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and μ.

(1 mark)

(d) Given that $\overrightarrow{OX} = \lambda\overrightarrow{OR}$, find $\overrightarrow{OX}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and λ.

(1 mark)

(e) Find the value of λ and of μ.

(3 marks)

(f) Find the value of $\frac{QX}{XS}$.

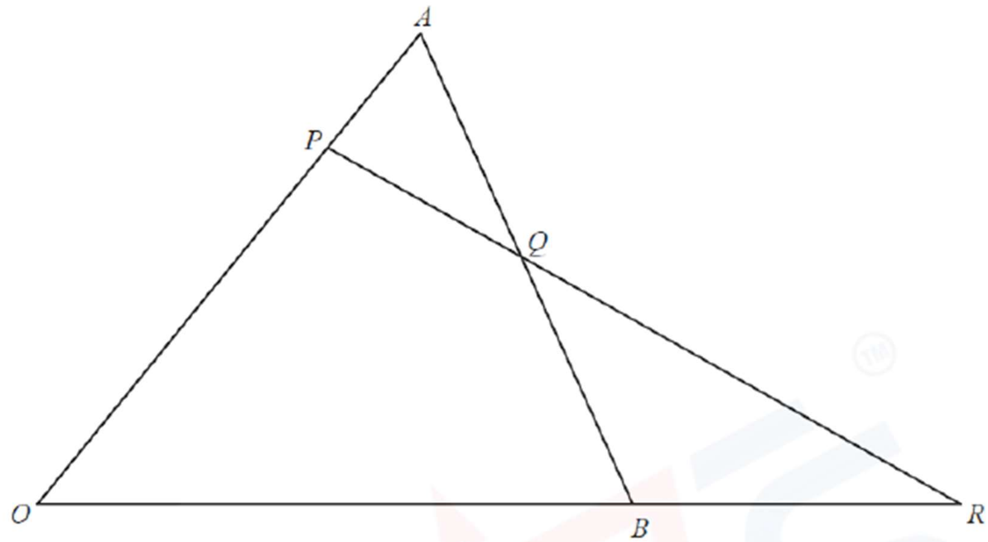
(1 mark)

(g) Find the value of $\frac{OR}{OX}$.

(1 mark)

26

(a)



The diagram shows a triangle OAB such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point P lies on OA such that $OP = \frac{3}{4}OA$. The point Q is the mid-point of AB. The lines OB and PQ are extended to meet at the point R. Find, in terms of $\mathbf{a}$ and $\mathbf{b}$,

Find $\overrightarrow{AB}$

(1 mark)

(b) Find $\overrightarrow{PQ}$. Give your answer in its simplest form.

(3 marks)

(c) It is given that $n\overrightarrow{PQ} = \overrightarrow{QR}$ and $\overrightarrow{BR} = k\mathbf{b}$, where n and k are positive constants.

Find $\overrightarrow{QR}$ in terms of n, $\mathbf{a}$ and $\mathbf{b}$.

(1 mark)

(d) Find $\overrightarrow{QR}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$.

(2 marks)

(e) Hence find the value of n and of k.

(3 marks)

27

(a) Relative to an origin O, the position vectors of the points A, B, C and D are

$$\overrightarrow{OA} = \begin{pmatrix} 6 \\ -5 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 10 \\ 3 \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} x \\ y \end{pmatrix} \text{ and } \overrightarrow{OD} = \begin{pmatrix} 12 \\ 7 \end{pmatrix}$$

Find the unit vector in the direction of $\overrightarrow{AB}$.

(3 marks)

(b) The point A is the mid-point of BC. Find the value of x and of y.

(2 marks)

(c) The point E lies on OD such that $OE : OD$ is $1 : 1 + \lambda$. Find the value of λ such that $\overrightarrow{BE}$ is parallel to the x-axis.

(3 marks)

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