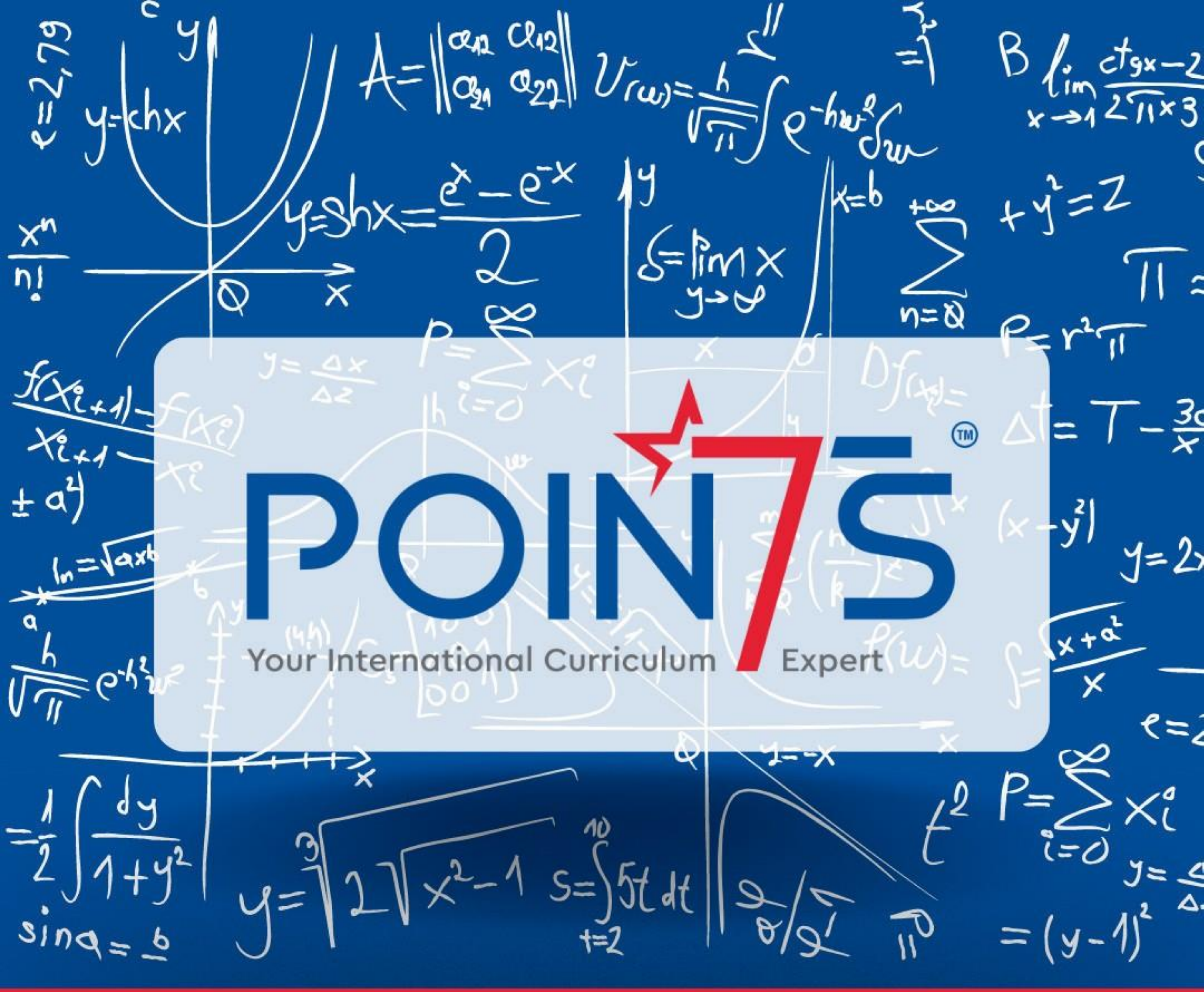

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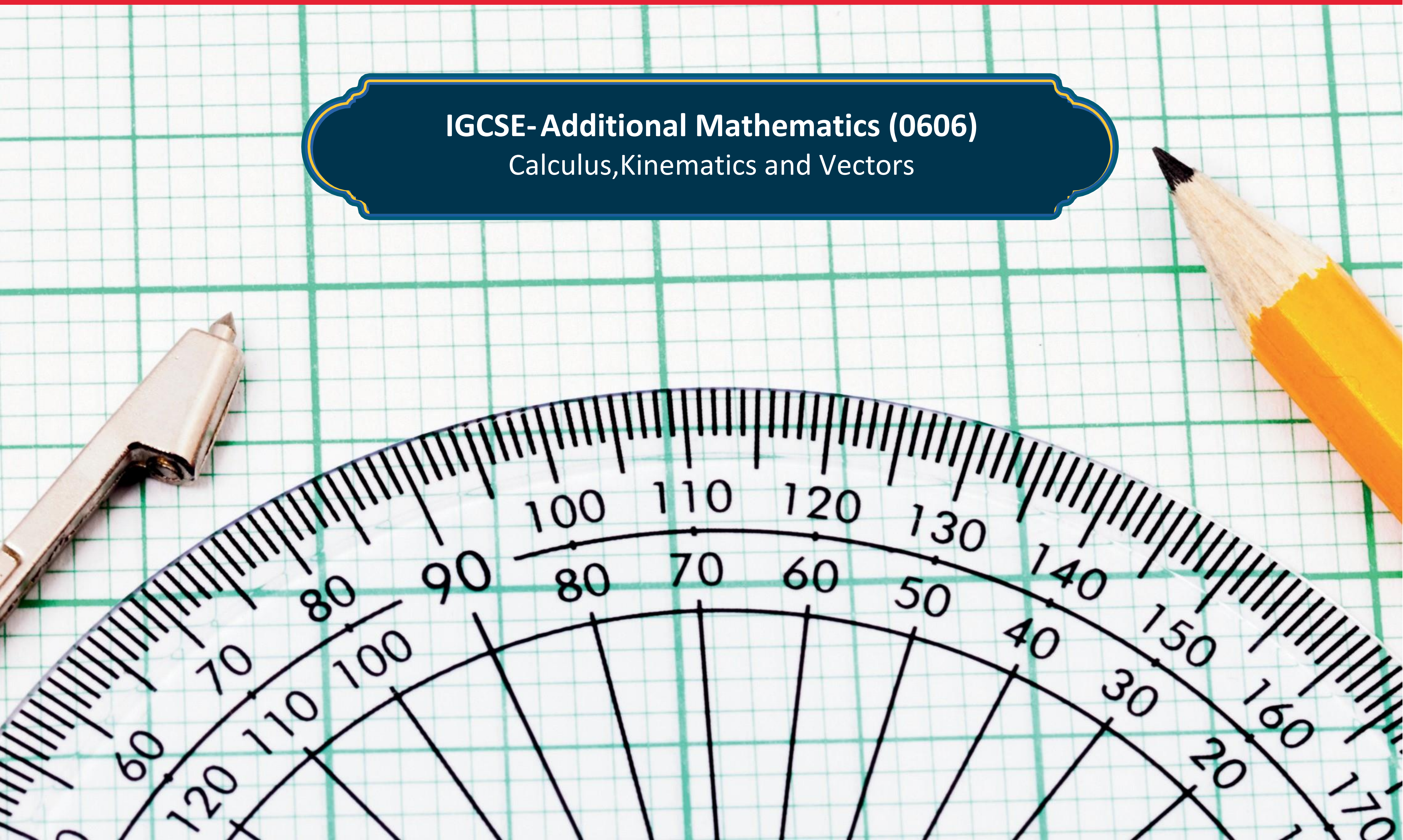
MATH TOPICAL WORKSHEETS



IGCSE- Additional Mathematics (0606)
Calculus,Kinematics and Vectors

1a

To add together two fractions, we need a common denominator.
Find the lowest common denominator by multiplying $2x - 3$ by $2x + 3$.

lowest common denominator is $(2x + 3)(2x - 3)$

Write each fraction with a common denominator.

$$\frac{3}{2x - 3} + \frac{3}{2x + 3} = \frac{3(2x + 3)}{(2x - 3)(2x + 3)} + \frac{3(2x - 3)}{(2x - 3)(2x + 3)}$$

Write as a single fraction.

$$\frac{3(2x + 3) + 3(2x - 3)}{(2x - 3)(2x + 3)}$$

[1]

Expand the brackets.

$$\frac{6x + 9 + 6x - 9}{4x^2 + 6x - 6x - 9}$$

Collecting like terms.

$$\frac{12x}{4x^2 - 9}$$ [1]

1b

Using part (a) to write the integral as two separate fractions.

$$\int \frac{3}{2x - 3} + \frac{3}{2x + 3} \mathrm{d}x$$

1 mark for each correct term [1]

Use the result $\int \frac{f'(x)}{f(x)} \mathrm{d}x = \ln|f(x)| + c$.

$$\frac{3}{2}\ln(2x - 3) + \frac{3}{2}\ln(2x + 3) + c$$

Use the logarithm law $\ln(m) + \ln(n) = \ln(m \times n)$.

$$\frac{3}{2}\ln(2x - 3)(2x + 3) + c$$

$$\frac{3}{2}\ln(4x^2 - 9) + c$$ [1]

1c

Using the answer to part (b).

$$\int_2^a \frac{12}{4x^2 - 9}\,dx = \left[\frac{3}{2}\ln(4x^2 - 9)\right]_2^a$$

Apply the limits.

$$\frac{3}{2}\ln(4a^2 - 9) - \frac{3}{2}\ln(4(2)^2 - 9) = \frac{3}{2}\ln(4a^2 - 9) - \frac{3}{2}\ln(7)$$

The right hand side of the equation, $\ln 5\sqrt{5}$ can be written as $\ln 5^{\frac{3}{2}}$.

Use the logarithm law $\ln(a)^m = m\ln(a)$.

$$\frac{3}{2}\ln(4a^2 - 9) - \frac{3}{2}\ln(7) = \frac{3}{2}\ln(5)$$

[1]

Collect like terms and divide each term by $\frac{3}{2}$.

$$\ln(4a^2 - 9) = \ln(5) + \ln(7)$$

Using the logarithm law $\ln(m) + \ln(n) = \ln(m \times n)$.

$$\ln(4a^2 - 9) = \ln(35)$$

Taking exponentials of both sides will cancel out the logarithm.

$$4a^2 - 9 = 35$$

$$4a^2 = 44$$

$$a^2 = 11$$

The question states that $a > 2$, therefore,

$a = \sqrt{11}$ [2]

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2a

Differentiate the equation using quotient rule and chain rule.

$$u = \ln(3x^2 - 5) \quad v = 2x + 1$$

$$\frac{du}{dx} = \left(\frac{1}{3x^2 - 5}\right) \times 6x \quad \frac{dv}{dx} = 2$$

correct $\frac{du}{dx}$ [1]

Use quotient rule, $\frac{dy}{dx} = \frac{(v)\left(\frac{du}{dx}\right) - (u)\left(\frac{dv}{dx}\right)}{v^2}$.

$$\frac{dy}{dx} = \frac{\left((2x+1)\left(\frac{6x}{3x^2-5}\right)\right) - (2)(\ln(3x^2 - 5))}{(2x+1)^2}$$

quotient rule [1]

all terms correct except $\frac{6x}{3x^2-5}$ [1]

When $x = \sqrt{2}$,

$$y = \frac{\ln(1)}{2\sqrt{2} + 1}$$

$\ln(1) = 0$, therefore when $x = \sqrt{2}$,

$$y = 0$$

[1]

Substitute $x = \sqrt{2}$ into $\frac{dy}{dx}$ to find the gradient of the curve at this point.

$$\frac{dy}{dx} = \frac{(2\sqrt{2}+1)\left(\frac{6\sqrt{2}}{3(\sqrt{2})^2 - 5}\right) - 2\ln(3(\sqrt{2})^2 - 5)}{(2\sqrt{2}+1)^2}$$

$$= \frac{\left(\frac{(2\sqrt{2}+1)(6\sqrt{2})}{1}\right) - 2\ln(1)}{(2\sqrt{2}+1)^2}$$

$$= \frac{(2\sqrt{2}+1)(6\sqrt{2})}{(2\sqrt{2}+1)^2}$$

$$= \frac{6\sqrt{2}}{2\sqrt{2}+1}$$

Take the negative reciprocal to find the gradient of the normal.

$$\text{gradient (normal)} = -\frac{2\sqrt{2}+1}{6\sqrt{2}}$$

Substitute this gradient and the point $(\sqrt{2}, 0)$ into the equation of line, $y - y_1 = m(x - x_1)$.

$$y - 0 = -\frac{2\sqrt{2}+1}{6\sqrt{2}}(x - \sqrt{2})$$

[1]

$$y = -\frac{(2\sqrt{2}+1)(x-\sqrt{2})}{6\sqrt{2}}$$

$$y = -0.451\ 184\ldots x + 0.638\ 071\ldots$$

$y = -0.451x + 0.638$ (3 s.f.) [1]

2b

As is small, use "change in y = gradient × change in x".

$$\text{change in } y = \frac{6\sqrt{2}}{2\sqrt{2}+1} \times h$$
$$= 2.216\ 388\ldots$$

The approximate change in y is $2.2h$ [1]

3a

Substitute $x = 1$ into the equation to find the y-coordinate.

$$y = 1^3 - 6(1)^2 + 3(1) + 10$$

$$y = 8$$

[1]

Differentiate the equation of the curve to find the gradient function.

$$\frac{dy}{dx} = 3x^2 - 12x + 3$$

[1]

Substitute $x = 1$ to find the gradient at this point.

$$\frac{dy}{dx} = 3(1)^2 - 12(1) + 3$$

$$\frac{dy}{dx} = -6$$

[1]

Find the equation of the tangent with point $(1, 8)$ and gradient -6.

$$y - 8 = -6(x - 1)$$

$$y - 8 = -6x + 6$$

$$y = -6x + 14$$

$y = -6x + 14$ [1]

3b

From part (a), the equation of the tangent is $y = -6x + 14$. Equate the tangent and the curve.

$$-6x + 14 = x^3 - 6x^2 + 3x + 10$$

Rearrange to get a cubic equation.

$$14 = x^3 - 6x^2 + 9x + 10$$

$$x^3 - 6x^2 + 9x - 4 = 0$$

[1]

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From part (a) we know $x = 1$ is a solution.

$(x-1)$ is a factor of the cubic

[1]

Factorise the cubic using long division (or inspection).

$$
\begin{array}{rrrrrl}
 & & 1x^2 & -5x & +4 & \\
\cline{2-5}
x-1 & \big)\,1x^3 & -6x^2 & +9x & -4 & \\
 & 1x^3 & -1x^2 & +0x & +0 & - \\
\cline{2-5}
 & & -5x^2 & +9x & -4 & \\
 & & -5x^2 & +5x & +0 & - \\
\cline{3-5}
 & & & 4x & -4 & \\
 & & & 4x & -4 & - \\
\cline{4-5}
 & & & & 0 &
\end{array}
$$

The factorised cubic is

$$(x-1)(x^2-5x+4)=0$$

[1]

Factorise the second bracket.

$$(x-1)(x-1)(x-4)=0$$

[1]

$(x-1)$ is a repeated factor and is the solution we already know from part (a). The other solution will give us the x-coordinate of where the tangent meets the curve again.

$$x-4=0$$

$$x=4$$

Substitute $x=4$ into the equation of the tangent to find the y-coordinate.

$$y=-6(4)+14=-10$$

The tangent and curve meet again at the point $(4,\ -10)$ [1]

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4a

This function is in the form $y = \frac{u}{v}$ so use the quotient rule, $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$.

$$u = \tan 3x \qquad v = \sin x$$

$$\frac{du}{dx} = 3\sec^2 3x \qquad \frac{dv}{dx} = \cos x$$

for $3\sec^2 3x$ [1]

Substitute these into the quotient rule.

$$\frac{dy}{dx} = \frac{\sin x \times 3\sec^2 3x - \tan 3x \times \cos x}{\sin^2 x}$$

using the quotient rule [1]
all terms correct [1]

Use $\sec x = \frac{1}{\cos x}$ to evaluate $\sec^2 3x$.

$$\frac{dy}{dx} = \frac{\sin x \times 3\left(\frac{1}{\cos^2 3x}\right) - \tan 3x \times \cos x}{\sin^2 x}$$

Substitute $x = \frac{\pi}{3}$.

$\frac{dy}{dx} = 2\sqrt{3}$ [1]

4b

As h is small, use "change in y = gradient × change in x".

From part (a), at $x = \frac{\pi}{3}$, $\frac{dy}{dx} = 2\sqrt{3}$.

$$\text{change in } y = 2\sqrt{3} \times h = 3.464\,101...$$

The approximate change in y is $3.5h$ [1]

4c

This is a connected rates of change question, so use chain rule.

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

The question states that x is increasing at the rate of 3 units per second.

$$\frac{dx}{dt} = 3$$

From part (a), at $x = \frac{\pi}{3}$.

$$\frac{dy}{dx} = 2\sqrt{3}$$

[1]

$$\therefore \frac{dy}{dt} = 2\sqrt{3} \times 3$$

$$\frac{dy}{dt} = 6\sqrt{3}$$ [1]

5a

Velocity is the change in speed with respect to time, therefore differentiate to get

$$v = \frac{ds}{dt} = 2e^{2t} - 10e^{t} - 12$$

$2e^{2t}$ [1]

fully correct expression [1]

Acceleration is the change in velocity with respect to time, therefore we differentiate to get

$$a = \frac{dv}{dt} = 4e^{2t} - 10e^{t}$$ [1]

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5b

If the particle is at rest, this means that the velocity is 0.

$$0 = 2e^{2t} - 10e^{t} - 12$$

This is a hidden quadratic which is easier to solve if we let $e^{t} = x$

$$0 = 2x^2 - 10x - 12$$

Dividing through by 2 gives

$$0 = x^2 - 5x - 6$$

We can factorise this by looking for a factor pair of -6 that has a sum of -5. This would be -6 and +1.

$$0 = (x - 6)(x + 1)$$

[1]

This means that the solutions are

$$x = 6 \Rightarrow e^{t} = 6$$

[1]

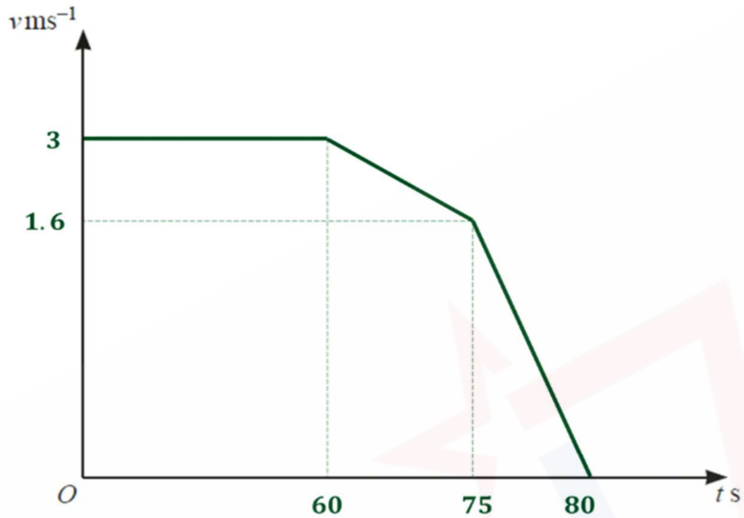
$x = -1 \Rightarrow e^{t} = -1$ which has no solutions

Taking natural log of both sides of the first solution gives

$t = \ln 6$ (seconds) [1]

6a

If the graph is travelling at a constant velocity, this would be a horizontal line on the graph. A deceleration would have a negative gradient. The final section of the graph should have a steeper negative gradient as the particle has stopped after 5 seconds.



correct shape with three distinct sections [1]

3 and 1.6 on vertical axis [1]

60, 75 and 80 on horizontal axis [1]

6b

The distance is the area under the graph.
Finding the area of the first section of the graph, the rectangle

$$60 \times 3 = 180$$

Finding the area of the second section of the graph, the trapezium

$$\frac{1}{2}(3 + 1.6) \times 15 = 34.5$$

Finding the area of the third section of the graph, the triangle

$$\frac{1}{2} \times 1.6 \times 5 = 4$$

attempting at least two areas [1]

all areas calculated correctly [1]

Finding the sum of these three areas

218.5 m [1]

6c

To find the deceleration, calculate the gradient of the line

$$\frac{1.6}{5}$$

$0.32\ \text{ms}^{-2}$ [1]

7a

We know that the curved surface area of a cone, S, is $\pi r l$.
r in this case is actually x and the slant height l is given as $\sqrt{x^2+y^2}$. But the expression given for the curved surface are is in terms of x only.
Therefore we need to find an expression for y in terms of x.

We are given a value for the volume, '10π'. Therefore equate 10π to the formula for volume of a cone in terms of the radius x and height y.

$$10\pi = \frac{1}{3}\pi x^2 y$$

Rearrange to get an expression for y in terms of x.

$$y = \frac{10\pi}{\frac{1}{3}\pi x^2}$$

$$= \frac{30}{x^2}$$

[1]

Now substitute $x = r$ and $l = \sqrt{x^2+y^2}$ where $y = \frac{30}{x^2}$ into $S = \pi r l$.

$$S = \pi x \sqrt{x^2 + \left(\frac{30}{x^2}\right)^2}$$

[1]

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This is starting to approach something that looks like the final answer. To get to the final answer, start by expanding the bracket inside the square root.

$$S = \pi x\sqrt{x^2 + \frac{900}{x^4}}$$

Note that there's no fraction inside the square root of the final answer. Rewrite '$x^2 + \frac{900}{x^4}$' as a single fraction.

$$S = \pi x\sqrt{\frac{x^6 + 900}{x^4}}$$

Now take the square root of the denominator of the fraction.

$$S = \pi x\frac{\sqrt{x^6 + 900}}{x^2}$$

Cancel the x and the x^2.

$$S = \pi \cancel{x}\frac{\sqrt{x^6 + 900}}{\cancel{x^2}}$$

$$S = \frac{\pi\sqrt{x^6 + 900}}{x}$$ [1]

7b

We will need to differentiate S and then equate the derivative to 0 to find the minimum value of x.

As differentiation will be involved, rewrite the square root as a fractional power.

$$S = \frac{\pi\sqrt{x^6 + 900}}{x} = \frac{\pi(x^6 + 900)^{\frac{1}{2}}}{x}$$

To differentiate S we need to use the quotient rule.

$$S = \frac{\pi(x^6 + 900)^{\frac{1}{2}}}{x} = \frac{u}{v}, \text{ where } u = \pi(x^6 + 900)^{\frac{1}{2}} \text{ and } v = x$$

From here we need to find $\frac{du}{dx}$ and $\frac{dv}{dx}$. However we need to use the chain rule to find $\frac{du}{dx}$.

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$$u = \pi(x^6+900)^{\frac{1}{2}}, \text{ let } u = \pi w^{\frac{1}{2}} \text{ and } w = x^6+900$$

Now differentiate *u* and *w*.

$$\frac{du}{dw} = \pi \times \frac{1}{2} w^{-\frac{1}{2}}, \quad \frac{dw}{dx} = 6x^5$$

And apply the chain rule, in this case $\frac{du}{dx} = \frac{du}{dw} \times \frac{dw}{dx}$.

$$\frac{du}{dx} = \pi \frac{1}{2} w^{-\frac{1}{2}} \times 6x^5$$

$$= \pi \frac{1}{2}(x^6+900)^{-\frac{1}{2}} \times 6x^5$$

$\pi \frac{1}{2}(x^6+900)^{-\frac{1}{2}}$ [1]

$$= 3\pi x^5(x^6+900)^{-\frac{1}{2}}$$

[1]

Now we are ready to go back to $S = \frac{u}{v}$ where $u = \pi\sqrt{x^6+900}$ and $v = x$.

$$u = \pi(x^6+900)^{\frac{1}{2}} \quad \text{and} \quad v = x$$

$$\frac{du}{dx} = 3\pi x^5(x^6+900)^{-\frac{1}{2}} \quad \frac{dv}{dx} = 1$$

Apply the quotient rule, $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$.

$$\frac{dS}{dx} = \frac{x\left(3\pi x^5(x^6+900)^{-\frac{1}{2}}\right) - \pi(x^6+900)^{\frac{1}{2}}}{x^2}$$

[1]

At the minimum value of S, $\frac{dS}{dx} = 0$, so equate this derivative to 0.

$$\frac{x\left(3\pi x^5(x^6+900)^{-\frac{1}{2}}\right) - \pi(x^6+900)^{\frac{1}{2}}}{x^2} = 0$$

Solve - start by multiplying by the denominator, x^2.

$$x\left(3\pi x^5(x^6+900)^{-\frac{1}{2}}\right)-\pi(x^6+900)^{\frac{1}{2}}=0$$

[1]

Simplify.

$$3\pi x^6(x^6+900)^{-\frac{1}{2}}-\pi(x^6+900)^{\frac{1}{2}}=0$$

Remove the negative power.

$$\frac{3\pi x^6}{(x^6+900)^{\frac{1}{2}}}-\pi(x^6+900)^{\frac{1}{2}}=0$$

$$\text{or}\quad \frac{3\pi x^6}{\sqrt{x^6+900}}-\pi\sqrt{x^6+900}=0$$

Multiply by $\sqrt{x^6+900}$.

$$3\pi x^6-\pi(x^6+900)=0$$

Divide by π and complete the solution.

$$3x^6-(x^6+900)=0$$
$$3x^6-x^6-900=0$$
$$2x^6=900$$
$$x^6=450$$

$$x=\sqrt[6]{450}=2.768229457\ldots \text{ or } 2.77\ (3\text{ s.f.})$$ [1]

Note that x is a length therefore $x\neq-\sqrt[6]{450}$

8a

Differentiate using product rule, $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$.

$$u=x \quad v=\cos x$$

$$\frac{du}{dx}=1 \quad \frac{dv}{dx}=-\sin x$$

[1]

$$\frac{dy}{dx}=(x)(-\sin x)+(1)(\cos x)$$

$$\frac{dy}{dx}=-x\sin x+\cos x$$ [1]

8b

Find the gradient of the curve at the point $x = \pi$.

$$\frac{dy}{dx} = -\pi \sin \pi + \cos \pi$$
$$\frac{dy}{dx} = -1$$

[1]

The gradient of the normal is the negative reciprocal of this.

$$\therefore\ m = 1$$

[1]

Substitute $x = \pi$ into the equation of the curve to find the corresponding y value.

$$y = \pi \cos \pi$$
$$y = -\pi$$

The normal passes through the point

$$(\pi, -\pi)$$

[1]

We know the gradient and a point the normal passes through so use the point-gradient form for the equation of a line.

$$y - (-\pi) = 1(x - \pi)$$

Write in the form $y = mx + c$ as required for the final answer.

$y = x - 2\pi$ [1]

9a

Use the product rule, $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$, and the chain rule.

Let $u = x^2 - 1$ and $v = (5x+2)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \left((x^2-1) \times \frac{1}{2}(5x+2)^{-\frac{1}{2}} \times 5\right) + \left((5x+2)^{\frac{1}{2}} \times 2x\right)$$

for $\frac{5}{2}(5x+2)^{-\frac{1}{2}}$ [1]

for differentiation of a product [1]

fully correct differentiation [1]

Simplify each bracket.

$$\frac{dy}{dx} = \left(\frac{5}{2}(x^2-1)(5x+2)^{-\frac{1}{2}}\right) + \left(2x(5x+2)^{\frac{1}{2}}\right)$$

Take out common factor of $\frac{1}{2}(5x+2)^{-\frac{1}{2}}$.

$$\frac{dy}{dx} = \frac{1}{2}(5x+2)^{-\frac{1}{2}}(5(x^2-1)+4x(5x+2))$$

[1]

Expand and simplify, writing as a fraction.

$$\frac{dy}{dx} = \frac{5x^2-5+20x^2+8x}{2\sqrt{5x+2}}$$

Simplify.

$$\frac{dy}{dx} = \frac{25x^2+8x-5}{2\sqrt{5x+2}}$$ [1]

9b

From part (a), $\frac{dy}{dx} = \frac{25x^2 + 8x - 5}{2\sqrt{5x+2}}$.

Stationary points occur when $\frac{dy}{dx} = 0$.

$$\frac{25x^2 + 8x - 5}{2\sqrt{5x+2}} = 0$$
$$25x^2 + 8x - 5 = 0$$

[1]

Use the quadratic formula (and/or calculator) to solve the quadratic equation.

$$x = \frac{-8 \pm \sqrt{8^2 - 4(25)(-5)}}{2(25)}$$

$$x = 0.3149... \text{ or } -0.6349...$$

$$x > 0 \text{ so } x = 0.3149...$$

Find y by substituting x into the equation of the curve.

$$y = (x^2 - 1)\sqrt{5x+2}$$
$$y = (0.3149...^2 - 1)\sqrt{5(0.3149...) + 2}$$
$$y = -1.7031...$$

$x = 0.315$ (3 s.f.) [1]

$y = -1.70$ (3 s.f) [1]

9c

Consider the gradient of the curve either side of the stationary point

From part (a), we know that $\frac{dy}{dx} = \frac{25x^2 + 8x - 5}{2\sqrt{5x+2}}$

The stationary point occurs at $(0.315, -1.70)$ so look at the gradient either side of this point.

When $x = 0.3$,

$$\frac{dy}{dx} = \frac{25(0.3)^2 + 8(0.3) - 5}{2\sqrt{5(0.3)+2}}$$

$$\frac{dy}{dx} = -0.093\ldots < 0$$

When $x = 0.4$,

$$\frac{dy}{dx} = \frac{25(0.4)^2 + 8(0.4) - 5}{2\sqrt{5(0.4)+2}}$$

$$\frac{dy}{dx} = 0.55 > 0$$

[1]

The gradient changes from positive to negative through the point $(0.315, -1.70)$ so the curve is " U -shaped".

$\therefore$ $(0.315, -1.70)$ is a minimum point [1]

10a

Write the two fractions as one with a single denominator of $(x+1)(3x+10)$.

$$\frac{1(3x+10) + 2(x+1)}{(x+1)(3x+10)}$$

Expand and simplify.

$$= \frac{(3x+10) + 2(x+1)}{(x+1)(3x+10)}$$

$$= \frac{3x + 2x + 10 + 2}{3x^2 + 10x + 3x + 10}$$

$$= \frac{5x + 12}{3x^2 + 13x + 10}$$

[1]

10b

Find the y-intercept of the line, point P, by substituting $x = 0$ into the equation of the curve.

$$y = \frac{5(0) + 12}{3(0)^2 + 13(0) + 10}$$
$$y = \frac{12}{10}$$
$$y = \frac{6}{5}$$

Therefore,

$$P\left(0, \frac{6}{5}\right)$$

The line segment PQ has a gradient of 1, and a y-intercept of $\frac{6}{5}$. Substitute these values into the equation of line, $y = mx + c$, to find the equation of the line.

$$y = x + \frac{6}{5}$$

The line crosses the x-axis when $y = 0$.

$$0 = x + \frac{6}{5}$$
$$x = -\frac{6}{5}$$

Therefore,

$$Q\left(-\frac{6}{5}, 0\right)$$

for both points correct [1]

Find the area of the triangle PQO.

$$\text{Area triangle} = \frac{1}{2} \times \frac{6}{5} \times \frac{6}{5}$$
$$= \frac{18}{25}$$

Integrate the curve between 0 and 2 to find the area under the curve, using the (reverse of the) result in part (a).

$$\int_0^2 \left(\frac{1}{x+1} + \frac{2}{3x+10}\right) dx$$

using part (a) [1]

$$\int_0^2 \left(\frac{1}{x+1} + \frac{2}{3x+10}\right) dx = \left[\ln(x+1) + \frac{2}{3}\ln(3x+10)\right]_0^2$$

one for each term [2]

Apply the limits.

$$\left[\ln(3) + \frac{2}{3}\ln(16)\right] - \left[\ln(1) + \frac{2}{3}\ln(10)\right] = \ln(3) + \frac{2}{3}\ln(16) - \frac{2}{3}\ln(10)$$

[1]

Use the laws of logarithms to rewrite the first term, creating a factor of $\frac{2}{3}$ in every term, and then to combine all three terms as a single logarithm.

$$\frac{2}{3}\ln(3)^{\frac{3}{2}} + \frac{2}{3}\ln(16) - \frac{2}{3}\ln(10) = \frac{2}{3}\ln\left(3\sqrt{3}\right) + \frac{2}{3}\ln(16) - \frac{2}{3}\ln(10)$$

$$= \frac{2}{3}\left(\ln\left(\frac{48\sqrt{3}}{10}\right)\right)$$

rewriting the first term [1]
simplify to a single term [1]

Simplify so this part of the shaded area is in the correct form.

$$\frac{2}{3}\ln\left(\frac{24\sqrt{3}}{5}\right)$$

Add together the area of the triangle and the area under the curve.

$$\mathbf{\frac{18}{25} + \frac{2}{3}\ln\left(\frac{24\sqrt{3}}{5}\right)} \quad [1]$$

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11a

Take 10 out of the integral as a constant.

$$\int_0^4 \frac{10}{5x+2}\,dx = 10\int_0^4 \frac{1}{5x+2}\,dx$$

Integrate the fraction using reverse chain rule.

$$= 10\left[\frac{1}{5}\ln(5x+2)\right]_0^4$$

$k \times ln(5x+2)$ [1]

Take $\frac{1}{5}$ as a factor.

$$= 10 \times \frac{1}{5}\left[\ln(5x+2)\right]_0^4$$

$$= 2\left[\ln(5x+2)\right]_0^4$$

[1]

Apply the limits.

$$= 2\,(\ln 22 - \ln 2)$$

[1]

$$= 2\ln\frac{22}{2}$$

$$= 2\ln 11$$

$2\ln 11$ [1]

11b

Use the law of logarithms, $(a^m)^n = a^{mn}$.

$$\int_0^{\ln 2} (e^{4x+2})^2\,dx = \int_0^{\ln 2} (e^{2(4x+2)})\,dx$$

$$= \int_0^{\ln 2} e^{8x+4}\,dx$$

[1]

Integrate.

$$= \left[\frac{1}{8} e^{8x+4} \right]_0^{\ln 2}$$

[1]

Apply the limits.

$$= \frac{1}{8} e^{8 \ln 2 + 4} - \frac{1}{8} e^4$$

Factorise $\frac{1}{8}$.

$$= \frac{1}{8}\left(e^{8 \ln 2 + 4} - e^4\right)$$

[1]

Rewrite the first exponential term as a product of 2 terms.

$$= \frac{1}{8}\left(e^{8 \ln 2} \times e^4 - e^4\right)$$

$$= \frac{1}{8}\left(e^{\ln 2^8} \times e^4 - e^4\right)$$

[1]

The exponential and natural log are inverses so cancel each other out.

$$= \frac{1}{8}\left(256e^4 - e^4\right)$$

Factorise e^4.

$$= \frac{1}{8} e^4 (256 - 1)$$

$$= \frac{255}{8} e^4$$

$\frac{255}{8} e^4$ [1]

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12

Use the quotient rule, i.e. for $y = \frac{u}{v}$, $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$.

$$u = \ln(3x^2 - 1),\ v = x + 2$$

Use the chain rule to differentiate the log function.

$$\frac{du}{dx} = \frac{6x}{3x^2 - 1}$$

[1]

$$\frac{dv}{dx} = 1$$

Apply the quotient rule.

$$\frac{dy}{dx} = \frac{(x + 2)\frac{6x}{3x^2 - 1} - \ln(3x^2 - 1)}{(x + 2)^2}$$

[1]

The gradient at the point $x = 1$ is required.

$$\frac{dy}{dx} = \frac{9 - \ln 2}{9} = 0.922\,983\ldots$$

[1]

Substitute $x = 1$ to the original equation to find the y-coordinate.

$$y = 0.231\,049\ldots$$

[1]

Substitute these values into $y = mx + c$ to find the equation of the tangent.

$$0.231\,049\ldots = 0.922\,983\ldots \times 1 + c$$
$$c = -0.691\,934\ldots$$

[1]

Final answer requires values rounded to 3 decimal places.

$y = 0.923x - 0.692$ [1]

13a

We actually need to work with a 'third' cone - formed from the larger hollow cone described in the question but with a height that is determined by the (higher) water level. All three cones are similar and since the radius of the smaller cone is half its own height, the radius of the 'third' cone will be half its own height too.

$$\therefore R = \frac{1}{2}(w + 180)$$

[1]

The volume of the water can be found by subtracting the volume of the smaller cone from that of the 'third' cone.

$$V = \frac{1}{3}\pi\left(\frac{1}{2}(w + 180)\right)^2(w + 180) - \frac{1}{3}\pi(90)^2(180)$$

[1]

Simplify.

$$V = \frac{1}{3}\pi\left(\frac{1}{2}\right)^2(w + 180)^2(w + 180) - \frac{1}{3}\pi(90)^2(180)$$

$$V = \left(\frac{1}{3}\right)\left(\frac{1}{4}\right)\pi(w + 180)^3 - 486000\pi$$

$$V = \frac{\pi}{12}(w + 180)^3 - 486000\pi$$ [1]

13b

This is a connected rates of change question, we will need to use chain rule. At time t,

$$\frac{dw}{dt} = \frac{dw}{dV} \times \frac{dV}{dt}$$

The rate at which the volume (V) is changing with respect to the height of the water (w) is

$$\frac{dV}{dw} = 3\frac{\pi}{12}(w + 180)^2$$

[1]

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We require $\frac{dw}{dV}$.

$$\frac{dw}{dV} = \frac{1}{\frac{\pi}{4}(w+180)^2}$$

We know the rate at which water is poured into the container with respect to time.

$$\frac{dV}{dt} = 10000$$

Apply chain rule.

$$\frac{dw}{dt} = \frac{1}{\frac{\pi}{4}(w+180)^2} \times 10000$$

[1]

$$\frac{dw}{dt} = \frac{40000}{\pi(w+180)^2}$$

When $w = 10$,

$$\frac{dw}{dt} = \frac{40000}{\pi(10+180)^2}$$

[1]

Simplify and evaluate.

$$\frac{dw}{dt} = \frac{40000}{\pi(190)^2}$$

$$\frac{dw}{dt} = 0.3526\ldots$$

$\frac{dw}{dt} = 0.353 \text{ cm s}^{-1}$ (3 s.f.) [1]

14a

Differentiate $y = e^{(4x - x^2)}$ using the chain rule

i.e. substitute $y = e^u$ and $u = 4x - x^2$ into $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$\frac{dy}{dx} = e^u \times (4 - 2x)$$

[M1]

Substitute back in $u = 4x - x^2$ to get the first derivative in x

$$\frac{dy}{dx} = (4 - 2x)e^{(4x - x^2)}$$

Factorise out 2 from the first bracket

$$\frac{dy}{dx} = 2(2 - x)e^{(4x - x^2)}$$

[A1]

14b

The first derivative from part (a) says that if $y = e^{(4x - x^2)}$ then

$$\frac{dy}{dx} = 2(2-x)e^{(4x-x^2)}$$

Reverse the process to go from the first derivative back to the equation of the curve

$$\int 2(2-x)e^{(4x-x^2)}dx = e^{(4x-x^2)} + c$$

The integral in the question is almost the same as this, but needs rearranging first

$$\int \frac{(2-x)e^{4x}}{e^{x^2}}dx = \int (2-x)e^{4x-x^2}dx$$

$$= \frac{1}{2}\int 2(2-x)e^{4x-x^2}dx$$

Now substitute the reversed integral from above into the right-hand side here

$$\int \frac{(2-x)e^{4x}}{e^{x^2}}dx = \frac{1}{2}\left[e^{(4x-x^2)} + c\right]$$

Expand (you can rename the unknown constant $\frac{1}{2}c$ as c again)

$$\int \frac{(2-x)e^{4x}}{e^{x^2}}dx = \frac{1}{2}e^{(4x-x^2)} + c$$

[B1 B1]

14c

Differentiate $\frac{dy}{dx} = 2(2-x)e^{(4x-x^2)}$ again to get $\frac{d^2y}{dx^2}$ but this time using the product rule $\frac{du}{dx}v + u\frac{dv}{dx}$

First find the derivatives of $u = 2(2-x) = 4 - 2x$ and $v = e^{(4x-x^2)}$ (note that $\frac{dv}{dx}$ is the answer to part (a))

$$\frac{du}{dx} = -2$$

$$\frac{dv}{dx} = 2(2-x)e^{(4x-x^2)}$$

Then substitute these into $\frac{d^2y}{dx^2} = \frac{du}{dx}v + u\frac{dv}{dx}$

$$\frac{d^2y}{dx^2} = (-2)\times e^{(4x-x^2)} + (4-2x)\times\left[2(2-x)e^{(4x-x^2)}\right]$$

[M1]

Simplify

$$\frac{d^2y}{dx^2} = -2e^{(4x-x^2)} + 4(2-x)(2-x)e^{(4x-x^2)}$$

[A1]

Factorise out $e^{(4x-x^2)}$ because the answer must be written in the form $(p+qx+rx^2)y$ and $y = e^{(4x-x^2)}$

$$\frac{d^2y}{dx^2} = [-2+4(2-x)(2-x)]e^{(4x-x^2)}$$

[M1]

Expand and simplify inside the brackets

$$\frac{d^2y}{dx^2} = (-2+4(4-4x+x^2))e^{(4x-x^2)}$$

$$= (-2+16-16x+4x^2)e^{(4x-x^2)}$$

$$= (14-16x+4x^2)e^{(4x-x^2)}$$

The result is meant to have the form $\frac{d^2y}{dx^2} = (p+qx+rx^2)y$

This is true, as $y = e^{(4x-x^2)}$

$$\frac{d^2y}{dx^2} = (14-16x+4x^2)y$$

[A1]

15

Integrate $\sin\left(6x-\frac{\pi}{2}\right)$ using reverse chain rule to find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \int \sin\left(6x-\frac{\pi}{2}\right)dx$$

$$\frac{dy}{dx} = -\frac{\cos\left(6x-\frac{\pi}{2}\right)}{6} + C$$

[2]

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Substitute $\frac{dy}{dx} = \frac{1}{2}$ and the given x -coordinate value.

$$\frac{1}{2} = -\frac{\cos\left(6\left(\frac{\pi}{4}\right) - \frac{\pi}{2}\right)}{6} + C$$

$$\frac{1}{2} = -\frac{\cos\left(\frac{6\pi}{4} - \frac{\pi}{2}\right)}{6} + C$$

[1]

Rearrange and solve to find C.

$$C = \frac{1}{3}$$

Substitute into the equation for $\frac{dy}{dx}$.

$$\frac{dy}{dx} = -\frac{\cos\left(6x - \frac{\pi}{2}\right)}{6} + \frac{1}{3}$$

Integrate $\frac{dy}{dx}$ using reverse chain rule to find the equation of the curve.

$$y = \int\left(-\frac{\cos\left(6x - \frac{\pi}{2}\right)}{6} + \frac{1}{3}\right)dx$$

$$y = -\frac{\sin\left(6x - \frac{\pi}{2}\right)}{36} + \frac{1}{3}x + A$$

[2]

Substitute the given coordinate into the equation of the curve to work out A.

$$\frac{13\pi}{12} = -\frac{\sin\left(6\left(\frac{\pi}{4}\right) - \frac{\pi}{2}\right)}{36} + \frac{1}{3}\left(\frac{\pi}{4}\right) + A$$

[1]

$$A = \pi$$

Substitute into the equation of the curve.

$$y = -\frac{1}{36}\sin\left(6x - \frac{\pi}{2}\right) + \frac{1}{3}x + \pi$$ [1]

16a

Differentiate the two terms separately.

Differentiate $\tan(x+4)$ with respect to x.

$$\sec^2(x+4)$$

Differentiate $-3\sin x$ with respect to x.

$$-3\cos x$$

either correct [1]

Put both parts together.

$\sec^2(x+4) - 3\cos x$ [1]

16b

Differentiate $\frac{\ln(2x+5)}{2e^{3x}}$ using the quotient rule.

$u = \ln(2x+5)$ and $v = 2e^{3x}$

$\frac{du}{dx} = \frac{2}{2x+5}$ and $\frac{dv}{dx} = 6e^{3x}$

for $\frac{2}{2x+5}$ [1]

for $6e^{3x}$ [1]

$$\frac{dy}{dx} = \frac{\left(2e^{3x} \times \frac{2}{2x+5}\right) - \left(6e^{3x} \times \ln(2x+5)\right)}{(2e^{3x})^2}$$

[1]

$$\frac{dy}{dx} = \frac{\left(2e^{3x} \times \frac{2}{2x+5}\right) - \left(6e^{3x} \times \ln(2x+5)\right)}{4e^{6x}}$$

[1]

Substitute $x = 1$ into the equation for $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{\left(2e^3 \times \frac{2}{7}\right) - (6e^3 \times \ln 7)}{4e^6}$$

$$= -0.1382093041$$

As h is small, use "change in y = gradient $\times$ change in x".

change in $y = -0.1382093041 \times h$

[1]

The approximate change in y is $-0.14h$ [1]

Vectors

1a

Substitute $\mathbf{a} = \alpha\mathbf{i} + \mathbf{j}$ and $\mathbf{b} = 12\mathbf{i} + \beta\mathbf{j}$ into $4\mathbf{a} - \mathbf{b} = (\alpha + 3)\mathbf{i} - 2\mathbf{j}$.

$$4(\alpha\mathbf{i} + \mathbf{j}) - (12\mathbf{i} + \beta\mathbf{j}) = (\alpha + 3)\mathbf{i} - 2\mathbf{j}$$

Expand the brackets.

$$4\alpha\mathbf{i} + 4\mathbf{j} - 12\mathbf{i} - \beta\mathbf{j} = (\alpha + 3)\mathbf{i} - 2\mathbf{j}$$

Equate the coefficients of $\mathbf{i}$.

$$4\alpha - 12 = \alpha + 3$$

$$3\alpha = 15$$

$$\alpha = 5$$

Equate the coefficients of $\mathbf{j}$.

$$4 - \beta = -2$$

method for both α and β [1]

$$6 = \beta$$

$\alpha = 5$ [1]
$\beta = 6$ [1]

1b

First find $\mathbf{b} - 4\mathbf{a}$.

$$\mathbf{b} - 4\mathbf{a} = (12\mathbf{i} + \beta\mathbf{j}) - 4(\alpha\mathbf{i} + \mathbf{j})$$

Use $\alpha = 5$ and $\beta = 6$ from part (a).

$$\mathbf{b} - 4\mathbf{a} = (12\mathbf{i} + 6\mathbf{j}) - 4(5\mathbf{i} + \mathbf{j})$$
$$\mathbf{b} - 4\mathbf{a} = -8\mathbf{i} + 2\mathbf{j}$$

To find the unit vector, divide by the magnitude.

$$\sqrt{(-8)^2 + (2)^2} = \sqrt{68} = 2\sqrt{17}$$

[1]

$$\text{Unit vector} = \frac{1}{2\sqrt{17}}(-8\mathbf{i} + 2\mathbf{j})$$

Unit vector in the direction of $\mathbf{b} - 4\mathbf{a}$ is $\frac{\sqrt{17}}{17}(-4\mathbf{i} + 2\mathbf{j})$ [1]

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Vectors

2a

For every one second, person A moves $(3\mathbf{i}-\mathbf{j})$ metres

Add five lots of $(3\mathbf{i}-\mathbf{j})$ on to the original position $(-4\mathbf{i}+\mathbf{j})$

$$-4\mathbf{i}+\mathbf{j}+5(3\mathbf{i}-\mathbf{j})$$

Expand and collect each component

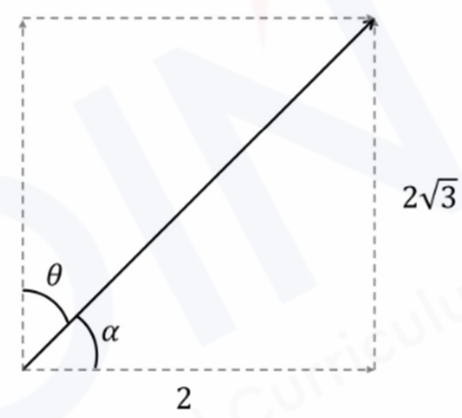
$$-4\mathbf{i}+\mathbf{j}+15\mathbf{i}-5\mathbf{j}$$

$(11\mathbf{i}-4\mathbf{j})$ **metres**

[B1]

2b

It helps to sketch the velocity vector and the bearing (measured clockwise from due north)



(i)

The speed is the magnitude of the vector (the hypotenuse of the triangle) so use Pythagoras' theorem

$$\sqrt{2^2+\left(2\sqrt{3}\right)^2}$$

[M1]

Vectors

Simplify

$$\sqrt{4+4\times3}=\sqrt{16}$$

4 metres per second

[A1]

(ii)

One way to find the bearing is to first find angle α (using trigonometry)

$$\tan\alpha=\frac{2\sqrt{3}}{2}=\sqrt{3}$$

[M1]

Solving this using exact trig values

$$\alpha=60°$$

[A1]

Hence find θ using $\theta=90°-\alpha$

$$\theta=30°$$

Write your final answer as a bearing (three digits needed)

030°

[A1]

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Vectors

3

Since $\overrightarrow{AC}$ is a multiple of $\overrightarrow{BC}$, we know that A, B and C must be collinear. Work out $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$\overrightarrow{AB} = \begin{pmatrix} 10 \\ -4 \end{pmatrix} - \begin{pmatrix} -5 \\ -7 \end{pmatrix}$$

$$\overrightarrow{AB} = \begin{pmatrix} 15 \\ 3 \end{pmatrix}$$

Use $\overrightarrow{AC} = 4\overrightarrow{BC}$ to find the ratio of AB to BC.

$$AB:BC = 3:1$$

[1]

$$\overrightarrow{BC} = \frac{1}{3}\overrightarrow{AB} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

Now find $\overrightarrow{OC}$.

$$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$$

$$\overrightarrow{OC} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ -4 \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 15 \\ -3 \end{pmatrix}$$

[1]

A unit vector in the direction of $\overrightarrow{OC}$ is required so we will need its magnitude.

$$|\overrightarrow{OC}| = \sqrt{15^2 + (-3)^2} = \sqrt{234} = 3\sqrt{26}$$

Find the unit vector by dividing $\overrightarrow{OC}$ by its magnitude.

$$\frac{1}{3\sqrt{26}}\begin{pmatrix} 15 \\ -3 \end{pmatrix} = \frac{1}{\sqrt{26}}\begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

$\frac{\sqrt{26}}{26}\begin{pmatrix} 5 \\ -1 \end{pmatrix}$ [1]

Vectors

4a

To find a unit vector, divide the vector by its magnitude.
Find the magnitude of the vector.

$$\text{Magnitude} = \sqrt{5^2 + (-12)^2}$$
$$\text{Magnitude} = 13$$

Divide the vector by 13.

$$\frac{1}{13}\begin{pmatrix} 5 \\ -12 \end{pmatrix} = \begin{pmatrix} \frac{5}{13} \\ -\frac{12}{13} \end{pmatrix}$$ [1]

4b

Multiply k and r by each component of the corresponding vectors.

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix} + \begin{pmatrix} -2k \\ 3k \end{pmatrix} = \begin{pmatrix} -10r \\ 5r \end{pmatrix}$$

Add together the vectors.

$$\begin{pmatrix} 4-2k \\ 1+3k \end{pmatrix} = \begin{pmatrix} -10r \\ 5r \end{pmatrix}$$

Equate the x components.

$$4 - 2k = -10r$$

Equate the y components.

$$1 + 3k = 5r$$

[1]

Solve the equations simultaneously.

$$\begin{array}{rl} & -4+2k=10r \\ - & 2+6k=10r \\ \hline & -6-4k=0 \\ & -4k=6 \end{array}$$

[1]

Vectors

$$k = -\frac{3}{2}$$

Substitute $k = -\frac{3}{2}$ into either of the equations to find r.

$$1 + 3\left(-\frac{3}{2}\right) = 5r$$
$$1 - \frac{9}{2} = 5r$$

$k = -\frac{3}{2}$, $r = -\frac{7}{10}$ [1]

4c

(i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$.

$$\overrightarrow{AB} = (3\mathbf{q} - \mathbf{p}) - \mathbf{p}$$
$$\overrightarrow{AB} = 3\mathbf{q} - 2\mathbf{p}$$

$3\mathbf{q} - 2\mathbf{p}$ [1]

(ii) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$.

$$\overrightarrow{AC} = (9\mathbf{q} - 5\mathbf{p}) - \mathbf{p}$$
$$\overrightarrow{AC} = 9\mathbf{q} - 6\mathbf{p}$$

$9\mathbf{q} - 6\mathbf{p}$ [1]

(iii) Both $\overrightarrow{AB}$ and $\overrightarrow{AC}$ share a common point, A, and $\overrightarrow{AC} = 3(\overrightarrow{AB})$.

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ share a common point – point A – and since $\overrightarrow{AC} = 3(\overrightarrow{AB})$, $\overrightarrow{AC}$ and $\overrightarrow{AB}$ are parallel [1]

(iv) Find $\overrightarrow{BC}$.

$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = \overrightarrow{OC} - \overrightarrow{OB}$.

$$\overrightarrow{BC} = (9\mathbf{q} - 5\mathbf{p}) - (3\mathbf{q} - \mathbf{p})$$
$$\overrightarrow{BC} = 6\mathbf{q} - 4\mathbf{p}$$
$$\overrightarrow{BC} = 2(3\mathbf{q} - 2\mathbf{p})$$

Therefore, $\overrightarrow{BC} = 2(\overrightarrow{AB})$ and the ratio of $AB:BC$ is $1:2$.

1:2 [1]

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Vectors

5a

(i)

E.g. travel from *A* to *O* to *B*

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$= \left(-\overrightarrow{OA}\right) + \overrightarrow{OB}$$
$$= -\mathbf{a} + \mathbf{b}$$

$\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$

[B1]

(ii)

Use your answer from part (i) to help

E.g. travel from O to *A* then *A* to *X* (which uses half of *A* to *B*)

$$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$$
$$= \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$$
$$= \mathbf{a} + \frac{1}{2}(-\mathbf{a} + \mathbf{b})$$

$\overrightarrow{OX} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

[B1]

(iii)

E.g. travel from C to O then O to *D*

$$\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD}$$
$$= \left(-\overrightarrow{OC}\right) + \overrightarrow{OD}$$
$$= -k\mathbf{a} + (-\mathbf{b})$$
$$= -k\mathbf{a} - \mathbf{b}$$

$\overrightarrow{CD} = -k\mathbf{a} - \mathbf{b}$

[B1]

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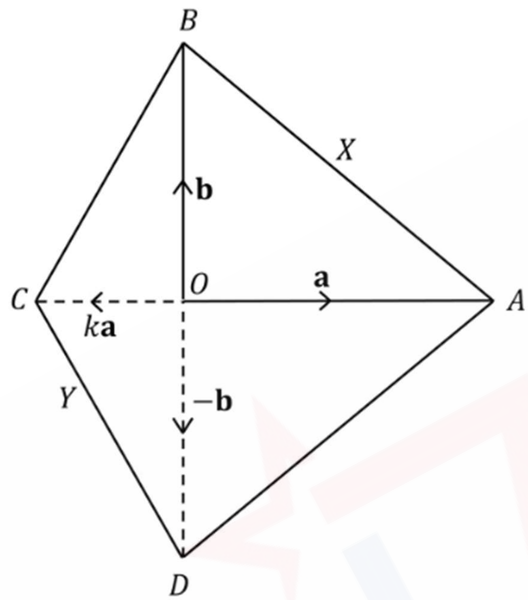
Vectors

5b

Add Y to the diagram



You found vector $\overrightarrow{CD} = -k\mathbf{a} - \mathbf{b}$ in part (a)(iii)

If Y splits the line CD into the ratio $1:2$ ($1 + 2 = 3$ parts to the ratio) then $\overrightarrow{YD}$ is $\frac{2}{3}$ of $\overrightarrow{CD}$

You found vector $\overrightarrow{CD} = -k\mathbf{a} - \mathbf{b}$ in part (a)(iii)

If Y splits the line CD into the ratio $1:2$ ($1 + 2 = 3$ parts to the ratio) then $\overrightarrow{YD}$ is $\frac{2}{3}$ of $\overrightarrow{CD}$

$$\overrightarrow{YD} = \frac{2}{3}\overrightarrow{CD}$$

$$= \frac{2}{3}(-k\mathbf{a} - \mathbf{b})$$

[B1]

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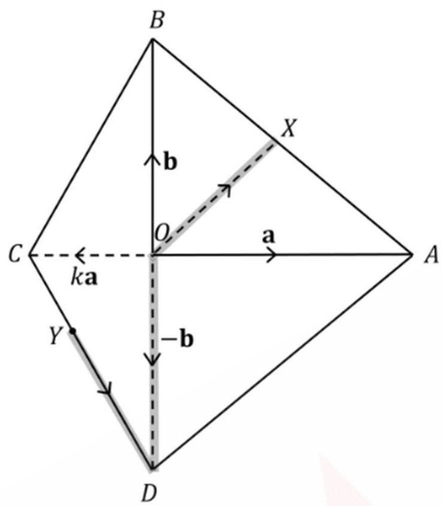
Now find $\overrightarrow{YX}$ e.g. by going from Y to D then D to O then O to X (using part (a)(ii))



$$\overrightarrow{YX} = \overrightarrow{YD} + \overrightarrow{DO} + \overrightarrow{OX}$$
$$= \overrightarrow{YD} + (-\overrightarrow{OD}) + \overrightarrow{OX}$$
$$= \frac{2}{3}(-k\mathbf{a} - \mathbf{b}) + (--\mathbf{b}) + \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$$

[M1]

Expand and simplify, collecting into $\mathbf{a}$ and $\mathbf{b}$ components

$$\overrightarrow{YX} = \left(-\frac{2}{3}k + \frac{1}{2}\right)\mathbf{a} + \left(-\frac{2}{3} + 1 + \frac{1}{2}\right)\mathbf{b}$$
$$= \left(-\frac{2}{3}k + \frac{1}{2}\right)\mathbf{a} + \frac{5}{6}\mathbf{b}$$

[A1]

If $\overrightarrow{YX}$ is meant to be parallel to $\mathbf{a} + \mathbf{b}$ then the $\mathbf{a}$ and $\mathbf{b}$ components in the expression above must be equal

$$-\frac{2}{3}k + \frac{1}{2} = \frac{5}{6}$$

Solve for k

$$-\frac{2}{3}k = \frac{1}{3}$$

$$k = -\frac{1}{2}$$

[A1]

Vectors

6a

Find $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$$

[1]

We are told that $AX:XB = 3:1$, therefore $\overrightarrow{AX} = \frac{3}{4}(\overrightarrow{AB})$.

$$\overrightarrow{AX} = \frac{3}{4}(\mathbf{b} - \mathbf{a})$$

Now we can find $\overrightarrow{OX}$.

$$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$$
$$\overrightarrow{OX} = \mathbf{a} + \frac{3}{4}(\mathbf{b} - \mathbf{a})$$

[1]

Expand and simplify.

$$\overrightarrow{OX} = \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$$ [1]

6b

B is the midpoint of OC. Therefore, $\overrightarrow{OC} = 2\boldsymbol{b}$.

$$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = \overrightarrow{OC} - \overrightarrow{OA}$$
$$\overrightarrow{AC} = 2\mathbf{b} - \mathbf{a}$$

$$\overrightarrow{AC} = 2\mathbf{b} - \mathbf{a}$$ [1]

6c

$\overrightarrow{AY} = \overrightarrow{AO} + \overrightarrow{OY}$. Since we are told that $\overrightarrow{OY} = h\overrightarrow{OX}$,

$$\overrightarrow{AY} = \overrightarrow{AO} + h\overrightarrow{OX} = h\overrightarrow{OX} - \overrightarrow{OA}$$
$$\overrightarrow{AY} = h\left(\frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}\right) - \mathbf{a}$$

$$\overrightarrow{AY} = \left(\frac{\mathbf{h} - 4}{4}\right)\mathbf{a} + \frac{3\mathbf{h}}{4}\mathbf{b}$$ [1]

Vectors

6d

Substitute $\overrightarrow{AY} = \left(\frac{h-4}{4}\right)\mathbf{a} + \frac{3h}{4}\mathbf{b}$ and $\overrightarrow{AC} = 2\boldsymbol{b} - \boldsymbol{a}$ into $\overrightarrow{AY} = m\overrightarrow{AC}$

$$\left(\frac{h-4}{4}\right)\mathbf{a} + \frac{3h}{4}\mathbf{b} = m(2\mathbf{b} - \mathbf{a})$$

Expand the brackets on the right-hand side.

$$\left(\frac{h-4}{4}\right)\mathbf{a} + \frac{3h}{4}\mathbf{b} = -m\mathbf{a} + 2m\mathbf{b}$$

[1]

Equate $\mathbf{a}$ and $\mathbf{b}$ vectors to form two equations in h and m.

$$\frac{h-4}{4} = -m$$

[1]

$$\frac{3h}{4} = 2m$$

[1]

Solve simultaneously.

$$m = \frac{3h}{8}$$

$$\therefore \frac{h-4}{4} = -\frac{3h}{8}$$

$$8h - 32 = -12h$$

$$20h = 32$$

$$h = \frac{32}{20} = \frac{8}{5}$$

Substitute into either of the original equations and solve for m.

$$\frac{3}{4}\left(\frac{8}{5}\right) = 2m$$

$$m = \frac{24}{40} = \frac{3}{5}$$

$h = \frac{8}{5}$ and $m = \frac{3}{5}$ [1]

Vectors

7a

Use "Velocity = Speed x Direction Unit Vector".

$$\text{Velocity} = 52 \times \frac{1}{\sqrt{(-5)^2 + 12^2}} \begin{pmatrix} -5 \\ 12 \end{pmatrix}$$

$$\text{Velocity} = \frac{52}{13} \begin{pmatrix} -5 \\ 12 \end{pmatrix}$$

$$\textbf{Velocity} = \begin{pmatrix} -20 \\ 48 \end{pmatrix} \text{km h}^{-1}$$ [1]

7b

The position vector of *P* at a time *t* hours after leaving *A* will be given by velocity x time (since *A* is the origin).

Use the velocity found in part (a).

Position vector of *P* at time *t* hours after leaving *A* is $\begin{pmatrix} -20t \\ 48t \end{pmatrix}$ km [1]

7c

Point *B* is **not** the origin so *B*'s initial position vector will need to be added to its "velocity vector x time".

$$\begin{pmatrix} -25t \\ 45t \end{pmatrix} + \begin{pmatrix} 12 \\ 8 \end{pmatrix}$$

The position vector of Q at time t is $\begin{pmatrix} 12 - 25t \\ 45t + 8 \end{pmatrix}$ km [1]

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Vectors

7d

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

Use the answers from parts (b) and (c).

$$\overrightarrow{PQ} = \begin{pmatrix} 12 - 25t \\ 45t + 8 \end{pmatrix} - \begin{pmatrix} -20t \\ 48t \end{pmatrix}$$

$$\overrightarrow{PQ} = \begin{pmatrix} 12 - 5t \\ 8 - 3t \end{pmatrix}$$ [1]

7e

Find, algebraically, the magnitude of your answer to part (d).

$$\left|\overrightarrow{PQ}\right| = \sqrt{(12 - 5t)^2 + (8 - 3t)^2}$$

[1]

$$= \sqrt{144 - 120t + 25t^2 + 64 - 48t + 9t^2}$$

$$\sqrt{34t^2 - 168t + 208}$$ [1]

7f

When P and Q are first 2 km apart, the magnitude of $\overrightarrow{PQ}$ will be 2.

$$\sqrt{34t^2 - 168t + 208} = 2$$

Solve, starting by squaring both sides.

$$34t^2 - 168t + 208 = 4$$

Make the quadratic equal to 0.

$$34t^2 - 168t + 204 = 0$$

[1]

Solve using the quadratic formula and/or calculator.

$$t = 2.148..., \ t = 2.792...$$

We want the *first* time that they are 2 km apart.

P and Q are first 2 km apart after 2.15 seconds (3 s.f.) [1]

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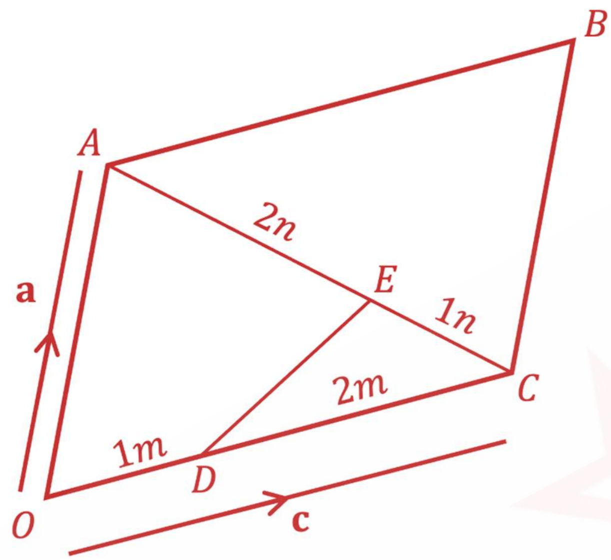
Vectors

Start by sketching a diagram with the information given. D is $\frac{1}{3}$ of the way from O to C and E is

8 $\frac{2}{3}$ of the way from A to C.



Write $\overrightarrow{OB}$ in terms of position vectors $\mathbf{a}$ and $\mathbf{c}$.

$$\overrightarrow{OB} = \mathbf{a} + \mathbf{c}$$

[1]

Write a path from D to E.

$$\overrightarrow{DE} = \overrightarrow{DC} + \overrightarrow{CE}$$

Now write $\overrightarrow{DC}$ and $\overrightarrow{CE}$ in terms of position vectors $\mathbf{a}$ and $\mathbf{c}$. Start with $\overrightarrow{DC}$.

$$\overrightarrow{DC} = \frac{2}{3}\mathbf{c}$$

[1]

Now look at $\overrightarrow{CE}$.

$$\overrightarrow{CE} = \frac{1}{3}\overrightarrow{CA}$$

Writing $\overrightarrow{CA}$ in terms of position vectors $\mathbf{a}$ and $\mathbf{c}$.

$$\overrightarrow{CA} = -\mathbf{c} + \mathbf{a}$$

Therefore,

$$\overrightarrow{CE} = \frac{1}{3}\overrightarrow{CA} = \frac{1}{3}(-\mathbf{c} + \mathbf{a})$$

[1]

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Vectors

Now write the path from D to E using the position vector expressions found above.

$$\overrightarrow{DE} = \overrightarrow{DC} + \overrightarrow{CE}$$
$$= \frac{2}{3}\mathbf{c} + \frac{1}{3}(-\mathbf{c} + \mathbf{a})$$

Simplify.

$$= \frac{2}{3}\mathbf{c} - \frac{1}{3}\mathbf{c} + \frac{1}{3}\mathbf{a}$$
$$= \frac{1}{3}\mathbf{c} + \frac{1}{3}\mathbf{a}$$

[1]

$$= \frac{1}{3}(\mathbf{c} + \mathbf{a})$$

Now we know that $\overrightarrow{OB} = \mathbf{c} + \mathbf{a}$ and $\overrightarrow{DE} = \frac{1}{3}(\mathbf{c} + \mathbf{a})$.

$$\overrightarrow{OB} = 3\overrightarrow{DE}$$

$k = 3$ [1]

Vectors

9a

Speed is given by the magnitude of the velocity so the first step is to work out the magnitude of the direction vector. The magnitude of vector $|a| = \left|\begin{pmatrix} x \\ y \end{pmatrix}\right|$ is given by $\sqrt{x^2 + y^2}$.

$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

[1]

We are told that the speed is 10 m s^{-1} so we want a velocity vector that is a scalar multiple of $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ but with a magnitude of 10. 10 is double 5 so we need to double the vector $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$, therefore the velocity vector is,

$$2 \times \begin{pmatrix} -4 \\ 3 \end{pmatrix} = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$$

[1]

We then need to multiply the velocity by time and add to the initial position vector to find the position vector P after *t* seconds, therefore

$$\begin{pmatrix} 30 \\ 10 \end{pmatrix} + \begin{pmatrix} -8 \\ 6 \end{pmatrix} t$$

The position vector of *P* after *t* seconds is $\begin{pmatrix} 30 - 8t \\ 6t + 10 \end{pmatrix}$ m [1]

9b

The speed is given by the magnitude of the velocity. Velocity is $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$. (The vector $\begin{pmatrix} -80 \\ 90 \end{pmatrix}$ is the initial position.)

$$\sqrt{5^2 + 12^2} = \sqrt{169}$$

13 m s^{-1} [1]

Vectors

9c

Substituting $t = 10$ into the position vectors for P and Q.

$$\overrightarrow{OP} = \begin{pmatrix} 30 \\ 10 \end{pmatrix} + \begin{pmatrix} -8 \\ 6 \end{pmatrix} \times 10$$

$$\overrightarrow{OP} = \begin{pmatrix} 30 \\ 10 \end{pmatrix} + \begin{pmatrix} -80 \\ 60 \end{pmatrix} = \begin{pmatrix} -50 \\ 70 \end{pmatrix}$$

$$\overrightarrow{OQ} = \begin{pmatrix} -80 \\ 90 \end{pmatrix} + \begin{pmatrix} 5 \\ 12 \end{pmatrix} \times 10$$

$$\overrightarrow{OQ} = \begin{pmatrix} -80 \\ 90 \end{pmatrix} + \begin{pmatrix} 50 \\ 120 \end{pmatrix} = \begin{pmatrix} -30 \\ 210 \end{pmatrix}$$

[1]

$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = \overrightarrow{OQ} - \overrightarrow{OP}$.

$$\begin{pmatrix} -30 \\ 210 \end{pmatrix} - \begin{pmatrix} -50 \\ 70 \end{pmatrix} = \begin{pmatrix} 20 \\ 140 \end{pmatrix}$$

The distance between two points is the magnitude of the vector between them.

$$\sqrt{20^2 + 140^2} = \sqrt{20000}$$

[1]

$100\sqrt{2}$ [1]

10a

$$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$$
$$\overrightarrow{OQ} = 2\mathbf{b} + \mathbf{a}$$

$\overrightarrow{OQ} = 2\mathbf{b} + \mathbf{a}$ [1]

10b

$$\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PO} + \overrightarrow{OS}$$
$$\overrightarrow{QS} = -\mathbf{a} - 2\mathbf{b} + 3\mathbf{a}$$
$$\overrightarrow{QS} = 2\mathbf{a} - 2\mathbf{b}$$

$\overrightarrow{QS} = 2\mathbf{a} - 2\mathbf{b}$ [1]

Vectors

10c

$$\overrightarrow{OX} = \overrightarrow{OP} + \overrightarrow{PQ} + \overrightarrow{QX}$$

The question tells us that $\overrightarrow{QX} = \mu\overrightarrow{QS}$.

$$\overrightarrow{OX} = 2\mathbf{b} + \mathbf{a} + \mu\overrightarrow{QS}$$

From part (b), $\overrightarrow{QS} = 2\mathbf{a} - 2\mathbf{b}$.

$$\overrightarrow{OX} = 2\mathbf{b} + \mathbf{a} + \mu(2\mathbf{a} - 2\mathbf{b})$$

$\overrightarrow{OX} = (1 + 2\mu)\mathbf{a} + (2 - 2\mu)\mathbf{b}$ [1]

10d

$$\overrightarrow{OX} = \lambda\overrightarrow{OR}$$

Find $\overrightarrow{OR}$.

$$\overrightarrow{OR} = \overrightarrow{OS} + \overrightarrow{SR}$$
$$\overrightarrow{OR} = 3\mathbf{a} + \mathbf{b}$$

Use this to find $\overrightarrow{OX}$.

$$\overrightarrow{OX} = \lambda(3\mathbf{a} + \mathbf{b})$$

$\overrightarrow{OX} = 3\lambda\mathbf{a} + \lambda\mathbf{b}$ [1]

10e

From part (c), $\overrightarrow{OX} = (1 + 2\mu)\mathbf{a} + (2 - 2\mu)\mathbf{b}$.

From part (d), $\overrightarrow{OX} = 3\lambda\mathbf{a} + \lambda\mathbf{b}$.

In the above equations, $\overrightarrow{OX}$ must be equivalent so equate both.

Compare coefficients of $\mathbf{b}$.

(1) $\quad 2 - 2\mu = \lambda$

Compare coefficients of $\mathbf{a}$.

(2) $\quad 1 + 2\mu = 3\lambda$

[1]

Vectors

Solve equations (1) and (2) simultaneously by adding them.

$$3 = 4\lambda$$

$$\lambda = \frac{3}{4}$$

for attempting to solve [1]

Substitute $\lambda = \frac{3}{4}$ into equation (1) to find μ.

$$2 - 2\mu = \frac{3}{4}$$

$$-2\mu = -\frac{5}{4}$$

$$\mu = \frac{5}{8}$$

$\lambda = \frac{3}{4}, \mu = \frac{5}{8}$ [1]

10f

$$\overrightarrow{QX} = \mu\overrightarrow{QS}$$

$\mu = \frac{5}{8}$ from part (e).

$$\overrightarrow{QX} = \frac{5}{8}\overrightarrow{QS}$$

Find the ratio of $QX:XS$.

$$QX:XS = 5:3$$

$$\frac{QX}{XS} = \frac{5}{3}$$

$\frac{QX}{XS} = \frac{5}{3}$ [1]

Vectors

10g

$$\overrightarrow{OR} = 3\mathbf{a} + \mathbf{b}$$

From part (d) we know that $\overrightarrow{OX} = 3\lambda\mathbf{a} + \lambda\mathbf{b}$.

From part (e) we know that $\lambda = \frac{3}{4}$.

$$\overrightarrow{OX} = \frac{9}{4}\mathbf{a} + \frac{3}{4}\mathbf{b} = \frac{3}{4}(3\mathbf{a} + \mathbf{b}) = \frac{3}{4}\overrightarrow{OR}$$

Find the ratio $OR:OX$.

$$OR:OX = 4:3$$

$$\frac{OR}{OX} = \frac{4}{3}$$

$\frac{OR}{OX} = \frac{4}{3}$ [1]

11a

The route from A to B, in known vectors, is via O.

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$\overrightarrow{OB} = \mathbf{b} - \mathbf{a}$ [1]

11b

$$\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ}$$

Since $OP = \frac{3}{4}OA$, we know that $OP:PA = 3:1$. Find $\overrightarrow{PA}$.

$$\overrightarrow{PA} = \frac{1}{4}\overrightarrow{OA} = \frac{1}{4}\mathbf{a}$$

[1]

Point Q is the midpoint of AB. Find $\overrightarrow{AQ}$.

$$\overrightarrow{AQ} = \frac{1}{2}\overrightarrow{AB}$$

Vectors

$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ from part (a).

$$\overrightarrow{AQ} = \frac{1}{2}(\mathbf{b} - \mathbf{a})$$

[1]

Add these together to find $\overrightarrow{PQ}$.

$$\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ}$$

$$\overrightarrow{PQ} = \frac{1}{4}\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$$

Expand and simplify.

$$\overrightarrow{PQ} = \frac{1}{4}\mathbf{a} + \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$$

$\overrightarrow{PQ} = \frac{1}{2}\mathbf{b} - \frac{1}{4}\mathbf{a}$ [1]

11c

$$\overrightarrow{QR} = n\overrightarrow{PQ}$$

From part (b) we know that $\overrightarrow{PQ} = \frac{1}{2}\mathbf{b} - \frac{1}{4}\mathbf{a}$.

$$\overrightarrow{QR} = n\left(\frac{1}{2}\mathbf{b} - \frac{1}{4}\mathbf{a}\right)$$

$\overrightarrow{QR} = n\left(\frac{1}{2}\mathbf{b} - \frac{1}{4}\mathbf{a}\right)$ [1]

Vectors

11d

To involve k, consider $\overrightarrow{QR}$ involving $\overrightarrow{BR}$.

$$\overrightarrow{QR} = \overrightarrow{QB} + \overrightarrow{BR}$$

Use $\overrightarrow{BR} = k\mathbf{b}$.

$$\overrightarrow{QR} = \overrightarrow{QB} + k\mathbf{b}$$

Find $\overrightarrow{QB}$.

$$\overrightarrow{QB} = \frac{1}{2}\overrightarrow{AB} = \frac{1}{2}(\mathbf{b} - \mathbf{a})$$

Find $\overrightarrow{QR}$.

$$\overrightarrow{QR} = \overrightarrow{QB} + k\mathbf{b}$$

[1]

$$\overrightarrow{QR} = \frac{1}{2}(\mathbf{b} - \mathbf{a}) + k\mathbf{b}$$

$$\overrightarrow{QR} = \left(\mathbf{k} + \frac{1}{2}\right)\mathbf{b} - \frac{1}{2}\mathbf{a}$$ [1]

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Vectors

11e

Parts (c) and (d) must be equal as both are $\overrightarrow{QR}$.

$$n\left(\frac{1}{2}\mathbf{b} - \frac{1}{4}\mathbf{a}\right) = \frac{1}{2}(\mathbf{b} - \mathbf{a}) + k\mathbf{b}$$

Equate coefficients of $\mathbf{b}$.

$$\frac{1}{2}n = \frac{1}{2} + k$$

Equate coefficients of $\mathbf{a}$.

$$-\frac{1}{4}n = -\frac{1}{2}$$

[1]

Solve the second equation to find n.

$$n = 2$$

Substitute $n = 2$ into the first equation to find k.

$$\frac{1}{2} \times 2 = \frac{1}{2} + k$$

$$k = \frac{1}{2}$$

$n = 2$ [1]

$k = \frac{1}{2}$ [1]

Vectors

12a

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$.

$$\overrightarrow{AB} = \begin{pmatrix} 10 \\ 3 \end{pmatrix} - \begin{pmatrix} 6 \\ -5 \end{pmatrix}$$

$$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$$

[1]

Work out the magnitude of $\overrightarrow{AB}$ using Pythagoras' Theorem.

$$\sqrt{4^2 + 8^2}$$

[1]

$$= \sqrt{80} = 4\sqrt{5}$$

Divide the vector by its magnitude and simplify.

$$\frac{1}{4\sqrt{5}}\begin{pmatrix} 4 \\ 8 \end{pmatrix} = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$\frac{\sqrt{5}}{5}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ [1]

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Vectors

12b

Because point *A* is a midpoint of *BC*,

$$\overrightarrow{OA} = \frac{1}{2} \times (\overrightarrow{OB} + \overrightarrow{OC})$$

$$\begin{pmatrix} 6 \\ -5 \end{pmatrix} = \frac{1}{2} \times \begin{pmatrix} 10+x \\ 3+y \end{pmatrix}$$

[1]

Multiply both sides by 2.

$$\begin{pmatrix} 12 \\ -10 \end{pmatrix} = \begin{pmatrix} 10+x \\ 3+y \end{pmatrix}$$

Therefore,

$$12 = 10 + x \quad -10 = 3 + y$$

$x = 2$, $y = -13$

both [1]

12c

$OE:OD = 1:1+\lambda$, therefore,

$$\frac{\overrightarrow{OE}}{\overrightarrow{OD}} = \frac{1}{1+\lambda}$$

Rearrange.

$$\overrightarrow{OE} = \frac{1}{1+\lambda}\overrightarrow{OD}$$

Substitute in the known values.

$$\overrightarrow{OE} = \frac{1}{1+\lambda}\begin{pmatrix} 12 \\ 7 \end{pmatrix}$$

[1]

Find $\overrightarrow{BE}$ in terms of λ.

$$\overrightarrow{BE} = \overrightarrow{OE} - \overrightarrow{OB}$$

$$\overrightarrow{BE} = \frac{1}{1+\lambda}\begin{pmatrix}12\\7\end{pmatrix} - \begin{pmatrix}10\\3\end{pmatrix}$$

$$\overrightarrow{BE} = \begin{pmatrix}\frac{12}{1+\lambda}\\\frac{7}{1+\lambda}\end{pmatrix} - \begin{pmatrix}10\\3\end{pmatrix}$$

$$\overrightarrow{BE} = \begin{pmatrix}\frac{12}{1+\lambda} - 10\\\frac{7}{1+\lambda} - 3\end{pmatrix}$$

Since $\overrightarrow{BE}$ is parallel to the x axis, the y component of the vector will be 0.

$$\frac{7}{1+\lambda} - 3 = 0$$

$$\frac{7}{1+\lambda} = 3$$

[1]

Rearrange and solve.

$$7 = 3(1+\lambda)$$

$$7 = 3 + 3\lambda$$

$$4 = 3\lambda$$

$\lambda = \frac{4}{3}$ [1]